One-Dimensional Motion: Displacement, Velocity, Acceleration

Physics 1425 Lecture 2
Today’s Topics

• The previous lecture covered measurement, units, accuracy, significant figures, estimation.
• Today we’ll focus on motion along a straight line: distance and displacement, average and instantaneous velocity and acceleration, the importance of sign.
• We’ll discuss the important constant-acceleration formulas.
Kinematics: Describing Motion

**Kinematics** describes *quantitatively* how a body moves through space. We’ll begin by treating the body as rigid and non-rotating, so we can fully describe the motion by following its center.

**Dynamics** accounts for the observed motion in terms of forces, etc. We’ll get to that later.
To measure motion, we must first measure position. We measure position relative to some fixed point \( O \), called the origin.

We give the ball’s location as \((x, y, z)\): we reach it from \( O \) by moving \( x \) meters along the \( x \)-axis, followed by \( y \) parallel to the \( y \)-axis and finally \( z \) parallel to the \( z \)-axis.

The frame can be envisioned as three meter sticks at right angles to each other, like the beginning of the frame of a structure.
One-Dimensional Motion: Distance Traveled and Displacement

- The frame of reference in one dimension is just a line!
- Think of a straight road.
- Driving a car, the distance traveled is what the odometer reads.
- The displacement is the difference $x_2 - x_1$ from where you started ($x_1$) to where you finished ($x_2$).
- They’re only the same if you only go in one direction!

This time we’ve made explicit that the $x$-axis also extends in the negative direction, so we can label all possible positions.
Distance and Displacement

- Take I-64 as straight, count Richmond direction as positive.
- Drive to Richmond: distance = 120 km (approx), displacement = 120 km.
- Drive to Richmond and half way back:
  - Distance = 180 km, displacement = 60 km.
- Drive to closest Skyline Drive entrance:
  - Distance = 35 km, displacement = -35 km.
Displacement is a Vector!

• A displacement along a straight line has magnitude and direction: + or −. That means it’s a vector.

• If the displacement $\Delta x = x_2 - x_1$, magnitude is written $|\Delta x| = |x_2 - x_1|$.

• Direction is indicated by attaching an arrowhead to the displacement:

Charlottesville to Richmond

Charlottesville to Skyline Drive
Average Speed and Average Velocity

• Average speed = distance car driven/time taken.
• Average velocity = displacement/time taken
  so average velocity is a vector! It can be negative.

• Formula for average velocity: \[ \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \]

• Example: round trip to Richmond.
  Average speed = 60 mph \approx 27 \text{ m/sec}.
  Average velocity = zero!
Instantaneous Velocity

• That’s the velocity at one moment of time: car speedometer gives instantaneous speed.

• To find this, need to find car’s displacement in a very short time interval (to minimize speed variation).

• Mathematically, we write: \[ v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}. \]

This “lim” just means taking a succession of shorter and shorter time intervals at the moment in time.
Average Trip Speed

You drive 60 miles at 60 mph, then 60 miles at 30 mph. What was your average speed?

A. 40 mph  
B. 45 mph  
C. 47.5 mph
Average Trip Speed

You drive 60 miles at 60 mph, then 60 miles at 30 mph. What was your average speed?

A. 40 mph
B. 45 mph
C. 47.5 mph

Total distance = 120 miles, time taken = 3 hours.
Acceleration

• Average acceleration = velocity change/time taken

\[ \bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \]

• Notice that acceleration relates to change in velocity exactly as velocity relates to change in displacement.

• Velocity is a vector, so acceleration is a vector.

• Taking displacement towards Richmond as positive:
  • Slowing down while driving to Richmond: negative acceleration.
  • Speeding up driving to Skyline Drive: also negative acceleration!
Instantaneous Acceleration

- This is just like the definition of instantaneous velocity:
- The instantaneous acceleration

\[ a = \lim_{{\Delta t \to 0}} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}. \]

- The acceleration at time \( t_1 \) is the slope of the velocity graph \( v(t) \) at that time.
Our Units for One-Dimensional Motion

- **Displacement**: meters (can be positive or negative)

- **Velocity** = rate of change of displacement, units: Meters per second, written $m/s$ or $m$.sec$^{-1}$.

- **Acceleration** = rate of change of velocity, units: Meters per second per second, written $m/s^2$ or $m$.sec$^{-2}$. 
Constant Acceleration

• Constant acceleration means the rate of change of velocity is constant.

\[ \frac{dv}{dt} = a = \text{constant.} \]

• The solution to this equation is

\[ v = v_0 + at. \]

• Check with an example: a car traveling at 10 m/s accelerates steadily at 2 m/s². How fast is it going after 2 secs? After 4 secs?
Distance Moved at Constant Acceleration

• At constant acceleration,

\[ \frac{dx}{dt} = v(t) = v_0 + at. \]

• The solution of this equation is

\[ x(t) = x_0 + v_0 t + \frac{1}{2} at^2. \]

• Here \( x_0 \) is the beginning position, \( v_0 \) the beginning velocity, \( a \) the constant acceleration.

• Exercise: check this by finding \( dx/dt \).
More about Constant Acceleration...

- At constant acceleration, the graph of velocity as a function of time $v(t) = v_0 + at$ is a straight line:

- If $v = v_0$ at $t = 0$, and $v = v_1$ at $t = t_1$, the average velocity over the time interval 0 to $t_1$ is

$$\bar{v} = \frac{v_0 + v_1}{2}.$$

- IMPORTANT! This formula is unlikely to be correct at nonconstant acceleration.
These formulas are worth memorizing: the last one is simply derived by eliminating $t$ between the first two.

\[
\begin{align*}
n &= n_0 + at \\
x &= x_0 + n_0 t + \frac{1}{2} at^2 \\
\bar{v} &= \frac{n_0 + v_1}{2} \\
\nu^2 &= n_0^2 + 2a (x - x_0)
\end{align*}
\]
The picture below shows time (4.56 secs) and speed (321 mph) for a standing start quarter mile at Indianapolis.
Assuming constant acceleration, what was the approximate horizontal g-force on the driver?

a. 0.3g  
b. 0.8g  
c. 1.5g  
d. 3g  
e. 5g