Freely Falling Objects

Physics 1425 Lecture 3
Today’s Topics

• In the previous lecture, we analyzed one-dimensional motion, defining displacement, velocity, and acceleration and finding formulas for motion at constant acceleration.

• Today we’ll apply those formulas to objects falling, but first we’ll review how we know that falling motion is at constant acceleration.
Galileo’s Idea

• Before Galileo, it was believed that falling objects quickly reached a natural speed, proportional to weight, then fell at that speed.

• Galileo argued that in fact falling objects continue to pick up speed (unless air resistance dominates) and that this acceleration is the same for all objects.

• But how to convince people? Watching a falling object, it’s all over so quickly.
Dropping a Brick

• Galileo claimed people already knew this without realizing it:

• Imagine driving a nail into a board by dropping a weight on it from various heights. Everyone already knows that the further it falls, the more impact—which must mean it’s moving faster.

• But how much faster? Not so easy to tell! Is there some way to slow down the motion?
Slowing down the motion...

• A feather falls slowly—but Galileo argued that that motion (fairly steady speed) was dominated by air resistance, so was unlike ordinary falling of a weighty object.

• He found another way to slow things down ... here’s his experiment—in two parts, the pendulum and the ramp.
A Two-Timing Pendulum

- Pendulum with peg

- First he took a pendulum swinging freely back and forth, then he introduced a fixed peg directly below the point the pendulum hangs from.
A Two-Timing Pendulum

- Pendulum with peg
- The pendulum will now move around a tighter arc on the right-hand side.
Clicker Question: Which is correct?

A. The pendulum is moving faster at the lowest point when it is coming in from the left (from the wider arc).
B. The pendulum is moving faster at the bottom when it is coming in from the right (from the tighter arc).
C. The pendulum speed at the bottom is the same either way.

(All neglecting the small effects of air resistance.)
Clicker Answer

The pendulum speed at the bottom is the same either way.

Because as it leaves the peg swinging back, it gets back to its original height, essentially, just as it did when the peg wasn’t there.

How high it gets obviously depends on how fast it’s moving at the bottom—so it must be the same in both cases.
Galileo’s Ramp Idea

• Galileo argued that his two-sided pendulum was like two ramps, one steep and one shallow, and a ball rolling to the bottom would have the same speed from either side.

And why not take one side **vertically** steep? Then the ball would just be falling!
Rolling Down the Ramp is Slow Mo Falling

• If rolling down the ramp the ball picks up the same speed that it would by just falling the same vertical distance, timing the slow roll can check Galileo’s claim that speed is picked up uniformly in falling!

• In particular, Galileo compared the times for the full distance roll and that for one-quarter of the full distance. We’ll do this.
Galileo’s Ramp Experiment Result

• Galileo found that in twice the time, the ball rolled four times the distance.

• This agrees with the constant acceleration formula for motion starting from rest at the origin:

\[ x = \frac{1}{2} at^2 \]

• He also checked many other distances and found good agreement.
Clicker Question

• Suppose in rolling down the ramp from rest at the top the ball is moving at 4 m/s at the bottom. What is its speed half way down the ramp?

A. 2 m/s
B. less than 2 m/s
C. more than 2 m/s.

(Neglect friction.)
Acceleration Due to Gravity \( g \)

- Having established that in the absence of air resistance all objects fall (near the Earth’s surface) with the same acceleration \( g \), \( g \) can be measured by timing a fall and using

\[
y = \frac{1}{2} gt^2.
\]

- Taking upwards as positive, velocity and position as functions of time will look like this:

\[
\begin{align*}
v(t) &= -gt \\
y(t) &= -\frac{1}{2}gt^2
\end{align*}
\]
Ball Thrown Vertically Upwards

- Having chosen **upwards as positive**, the acceleration $a = -g = -9.8 \text{ m/s}^2$.
- While the ball is moving upwards it is *losing speed* at this rate.

- The velocity/time graph:
  
  $$v(t) = v_0 - gt.$$  

- The slope of the line is the **acceleration** $a = -g$. 

![Velocity-time graph](image)
Clicker Question

A ball is thrown vertically upwards. What is the direction of its acceleration \textit{at the highest point} it reaches?

A. Downwards
B. Upwards
C. At that point, the acceleration is zero.
Clicker Answer

- The acceleration $a = -g = -9.8 \text{ m/s}^2$ at all times.

That includes the topmost point!

Remember the acceleration is the rate of change of velocity,

$$
\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}
$$

so even if $v = 0$ at some instant $t_1$, it isn’t zero any other point $t_2$. 

![Graph showing velocity over time with a downward slope](graph.png)