Galileo and Freely Falling Objects

Michael Fowler physics 142E Lec 3, Jan 14, 2009

Demolishing Traditional Beliefs

Before Galileo's time (around 1600) the authority on motion, and much else, was Aristotle. The problem with understanding falling motion is that it's over so fast it's difficult to observe, unless you drop something very light, like a feather, or drop something through liquid, water, say, or, even better, molasses. For these slow motions, it's pretty clear that after a brief initial period, the object falls at a steady rate. It was guessed—and accepted—that ordinary fast fall, like dropping a rock or a cannonball, was just a speeded up version of what was observed in slow fall: an initial rapid increase in speed, then steady motion. Furthermore, it was assumed that heavier things fall faster: certainly true of rocks and feathers, but a general belief in the elegance of natural laws led to the conclusion that the speed would be proportional to the weight, so a 2 kg rock would fall at twice the rate of a 1 kg rock. After all, if you hold a 2 kg rock, it seems more eager to get down than a 1 kg rock.

Galileo disposed of these traditional beliefs *without even doing an experiment*—here's his argument: first, assume it's true that the 2 kg rock falls twice as fast as the 1 kg rock. Suppose you join them with a string, and drop them. How fast will they fall? Well, the 2 kg rock will dominate, the 1 kg rock will be a drag, so no doubt the net rate of fall will be somewhat less than the 2 kg rock alone.

But now you shorten the string, pulling them together. That shouldn't really change things: except that now you have what amounts to a 3 kg rock—so it should be falling 50% faster than the 2 kg rock! Assuming speed of fall is proportional to weight has led to a contradiction!

Galileo next turned his attention to the claim that after brief initial acceleration, objects fell at constant speed. He pointed out that actually everyone knows instinctively this isn't true. Think of dropping a small rock on your foot. Probably from one centimeter above, it's not too painful. Even a few centimeters might be OK. But from a meter it could be nasty. Knowing this means we already know it's *continuing to pick up speed*! And Galileo went to further heights by considering the rock falling on a stake and driving it into the ground—the higher you drop it from, the further in it drives the stake, and therefore the faster it's going. It's picking up speed all the time.

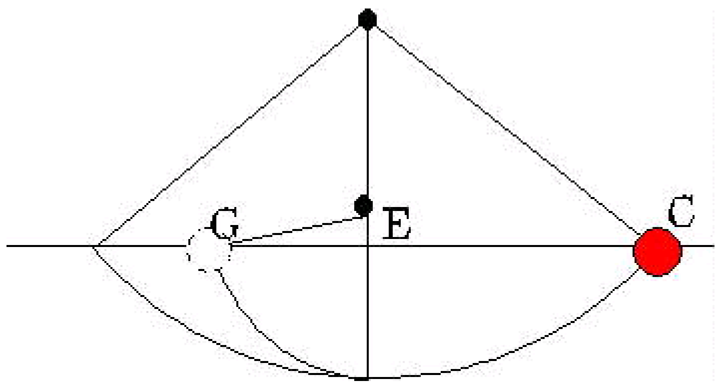
Galileo's New Idea: Constant Acceleration, not Constant Speed

But Galileo came up with a lot more: he claimed that, ignoring air resistance, **a falling object picks up speed at a steady rate—its downward velocity is just proportional to how long it's been falling.** Of course, air resistance itself depends on speed, so in a long fall, such as from an airplane, eventually it becomes important, but we're talking here about objects falling a few tens of meters at most.

But how could he possibly establish, with the available technology, this steady increase in speed? He needed to slow the motion somehow, without changing its nature—which air resistance and water resistance did, in his opinion.

Galileo Does Some Experiments

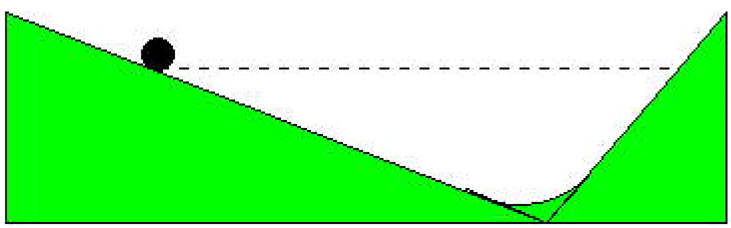
He started by thinking about a pendulum with a barrier, a peg that effectively shortens its length during half of the swing; here's his own diagram:



As the pendulum swings down from C, the string is caught on the peg at E and the pendulum swings around E to the point G.

He found that G was the same height as E (allowing for gradual slowing down) , and argued that the pendulum going down the shorter steeper path from G must pick up the same speed as it had falling from C, because it got back to C (or very nearly), just as it would have if the peg wasn't there, and it had gone up to the point opposite C instead of to G.

So what? He argued that the same thing would happen with a steep ramp connected to a gentler slope:



the speed at the bottom would be the same either way. Next, what if the steep slope is infinitely steep? Then the ball simply falls! So, the speed a ball picks up falling through a given distance is the same as it would pick up, in a longer time of course, rolling down a ramp.

So, if it can be established that a ball picks up speed at a steady rate as it rolls down a ramp, it picks up speed at a steady rate in simply falling.

Galileo carefully measured the time the ball took to roll down the ramp, then measured how long it took to roll one-quarter way down. He found that in twice the time the ball rolled four times as far, and this is exactly what you expect if the ball is picking up speed steadily: in twice the time, it's picked up twice the speed, so over the first two seconds (starting from rest) its average speed for the period is twice what it is over the first second.

So, ***it has twice the average speed, and rolls for twice the time—it goes four times as far***.

Having established that falling motion is uniformly accelerating, we're ready to use the formulas we worked out in the last lecture.

But What About Air Resistance?

Galileo claimed that a feather fell more slowly because of air resistance—but he couldn't prove it! We can: we can drop a feather in a vacuum. (The first vacuum pumps, constructed by Boyle and others, were about half a century after Galileo.) It's also been proven in a better vacuum environment: the surface of the Moon. And, gravity is a lot lower there, so it's easier to see what happens: the hammer and the feather fall at the same rate.

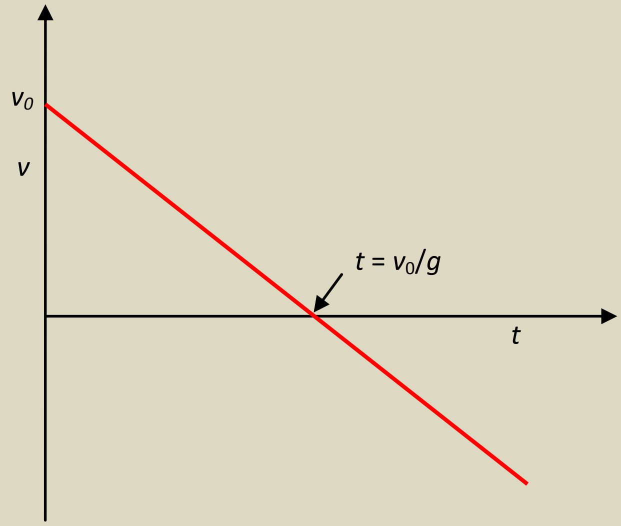
Air resistance to motion of a falling body increases approximately as the square of the velocity, so eventually balances the weight—this happens at speeds around 90 meters per sec for a skydiver with pulled in limbs. This means that at 9 meters per sec, which you would reach off a meter high wall, say, air resistance is only of order 1% of your weight.

Vertical Motion Under Gravity

Galileo's basic finding was that, neglecting air resistance, all falling objects pick up speed at the same steady rate, which we now know is 9.80 meters per second speed gain for each second of fall. We call this acceleration rate g.

He also observed that an object thrown vertically upwards loses speed at this same rate. But with our definition of acceleration as rate of change of velocity, and velocity, remember, can be positive or negative depending on which way the object is moving, we see that here again the acceleration is in the downward direction.

o, an object thrown vertically upwards has constant downward acceleration g throughout its flight. This is best visualize graphically:



Velocity as a function of time for an object thrown upwards: $v = v_0 - gt$

Remember the slope of the line in a velocity-time graph is the acceleration: this is constant and negative.

What about total distance traveled up to time *t*? That's just the area under the curve, as we discussed earlier. But now there's a new wrinkle: **area *below* the *x*-axis counts as *negative*.** This is obvious if you think for a moment: when the velocity goes negative, the object is coming back down, its distance from its initial point is decreasing.

The maximum upward distance from the initial point is when the area under the curve is a maximum, evidently the time when the velocity is zero—obviously, it stops at the top, and the time to get there is

$$\text{time to top } t = v_0 / g.$$

The area of the triangle under the curve, ½ base x height, is the maximum distance traveled upwards:

$$x = \tfrac{1}{2}\frac{v_0}{g}v_0 = \frac{v_0^2}{2g}.$$

(This is just equivalent to the formula $v^2 = 2ax$ derived previously, except that for the upward motion the velocity steadily *decreases* to zero—the subsequent downward motion takes the same time, always of course neglecting air resistance.)

The distance traveled upwards in terms of time taken is $x = \frac{v_0^2}{2g} = \frac{(gt)^2}{2g} = \tfrac{1}{2}gt^2.$

Putting in Some Numbers: Hang Time, Throw Speed and Height Reached

To get some feeling for vertical motion under gravity, without much numerical work, let's take

***g* = 10 m. sec^{-2} downwards** which is only 2% off.

Suppose I throw a ball upwards at 20 m per sec, and catch it when it comes down. How long was it in the air?

Answer: it accelerates downwards at 10 m per sec per sec, so its upwards velocity is 10 m per sec after one sec, 0 m per sec after two secs, -10 m per sec after three secs, -20 m per sec after 4 secs: it must be back at my hand at this point, since it's now going as fast downwards as it was initially thrown upwards.

So the time in the air—the **"hang time"**—is $2v_0/g$.

Notice that if you clock the hang time, you can figure out the initial vertical velocity: $v_0 = gt_{\text{hang}}/2$. (In fact, this formula gives the *vertical component* of the throw velocity even if the throw is not vertical, as we'll see in the next lecture.)

How high does a ball thrown upwards at 20 m per sec get?

Answer: We've established that it keeps moving upwards for 2 seconds, and its average upward speed is the average of its initial and final speeds, 20 m per sec and zero m per sec, that is, the average speed is 10 m per sec. therefore, in 2 seconds, it goes 20 meters.

Using the formula,

$$h = \tfrac{1}{2}gt_{\text{up}}^2 = \tfrac{1}{8}gt_{\text{hang}}^2 \cong 1.2t_{\text{hang}}^2 \text{ meters.}$$

So just by clocking the hang time, you can find initial upward velocity and how high it goes. Of course, this formula neglects air resistance, so isn't too good for a 100 mph tennis ball.

Cliff Edge Problem

I stand leaning slightly over the edge of a cliff with a vertical drop of 40 meters. I throw a ball vertically upwards at 10 meters per sec, so that as it comes back down it continues past me to the lower ground 40 meters below. How long is it in the air?

Answer: we need the distance/time formula, $x = x_0 + v_0t + \tfrac{1}{2}at^2$.

Taking x_0 = 0, $x(t)$ = -40 meters, counting *upwards* as *positive*, v_0 = 10 m per sec, and we'll take g = -10 m per sec per sec.

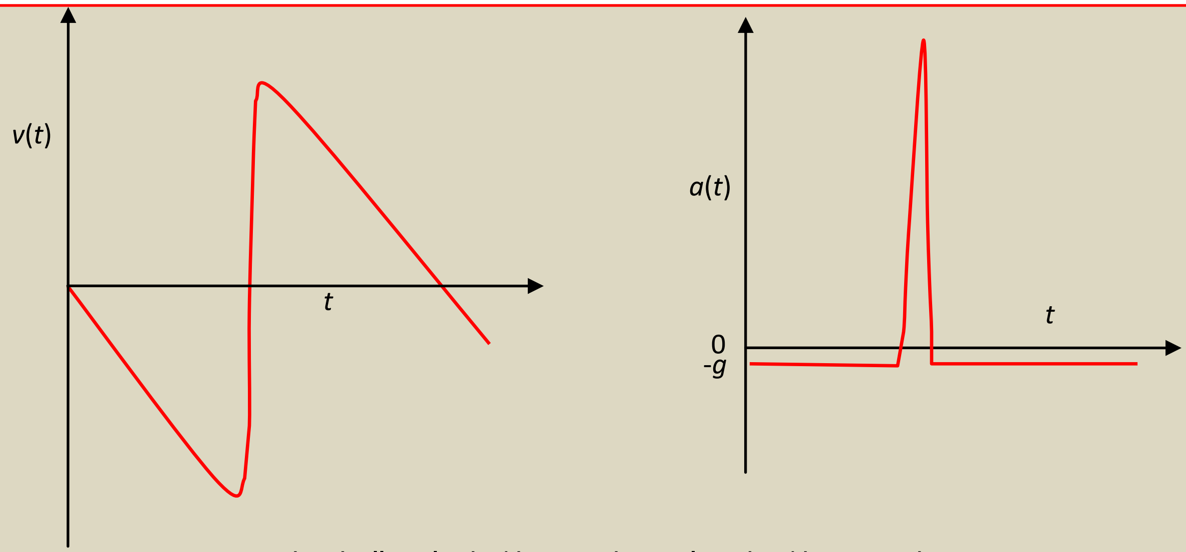
The equation becomes: $-40 = 10t + \tfrac{1}{2}10t^2$, or $5t^2 - 10t - 40 = 0$, easily solved by the usual quadratic formula to give the two roots $t = 4$, $t = -2$. (I picked easy numbers—usually this needs a calculator!)

What does the negative root, t = -2 mean? The equation doesn't know that the motion began at t = 0, it just knows the ball is moving up at 10 m per sec at that moment. But that could have happened if someone had thrown it up from the bottom of the cliff at 30 m per sec two seconds earlier, that is, at t = -2. Finding the time for a given height in a vertical gravity trajectory always gives a quadratic equation, even if the ball is initially thrown *downwards*—you need to use your knowledge of the actual situation to figure out which answer to choose. The two roots of the quadratic become equal if you choose the height to be the maximum reached.

Bouncing Ball

To understand acceleration better, it's worth thinking through a familiar situation where the acceleration changes: a bouncing ball. As the ball falls to the floor, the velocity increases at a steady rate, $-g$, of course, counting upwards as positive, but then *during the actual bounce* the velocity changes *very rapidly* from $-v$ to $+v$, the slope of the velocity/time plot is therefore steeply positive, that is, the acceleration is very large and positive. As soon as the bounce finishes, and the ball loses contact with the floor, the acceleration goes back to $-g$.

So the graphs look like this:



Bouncing ball: velocity/time and acceleration/time graphs

Exercise: recall the area under the velocity/time graph up to a certain time is the distance from the origin at that time. What does the area under the acceleration/time graph up to a certain time represent?