Using Newton's Laws

Michael Fowler Physics 142E Lec 7 January 29, 2009

The Laws

First, let's put the Laws together in one place:

1. If a body has zero total force acting on it, its velocity will not change.

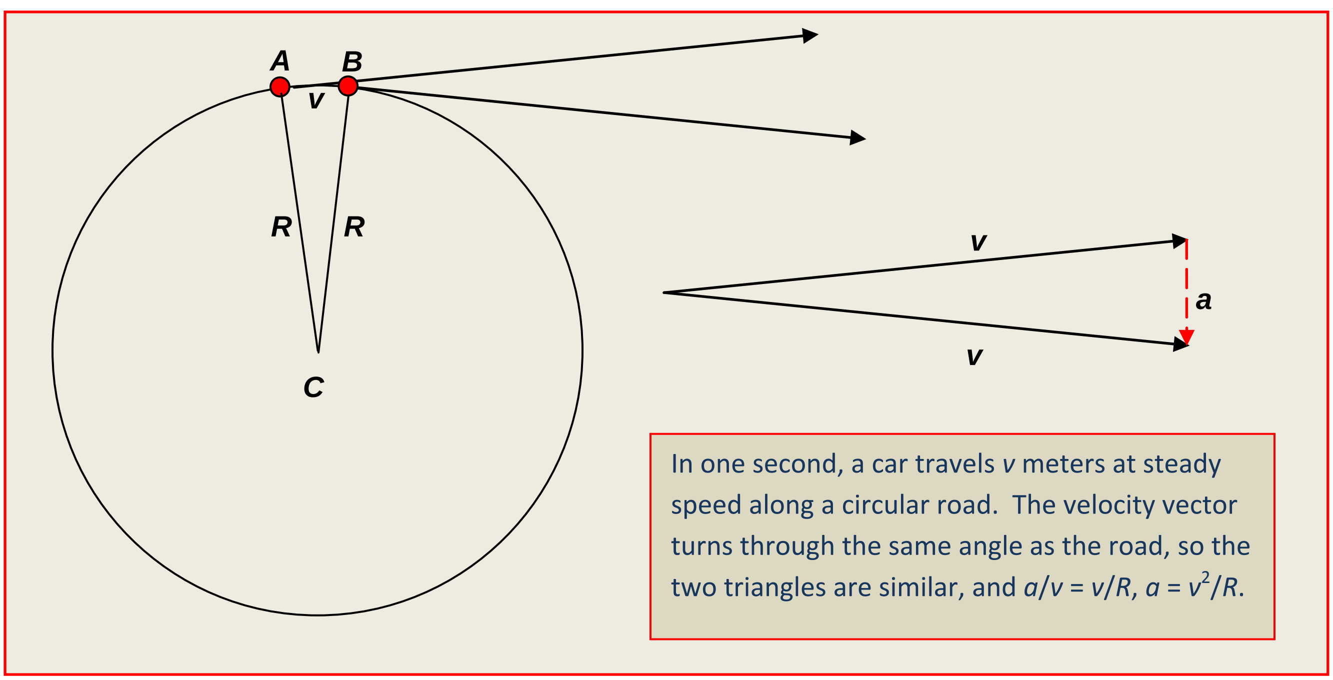
2. If a body of mass *m* has total force $\vec{F}$ acting on it, it will accelerate at $\vec{a}$, where $\vec{F} = m\vec{a}$.

3. Action = Reaction: if body *A* exerts force $\vec{F}$ on body *B*, then body *B* exerts force $-\vec{F}$ on body *A*.

We'll begin by focusing on the Second Law. It looks pretty straightforward, but in fact involves two subtle concepts: first, acceleration; second, it's not always easy to find the *total* force on a body.

Acceleration

The acceleration caused by stepping on the gas on a straight road is easy to understand, but it's not the whole story: acceleration is a vector, the rate of change of the velocity vector, and it can change just by changing direction at constant speed. The classic case is constant speed motion on a circular track: in fact, we already discussed that in talking about the Moon's acceleration towards the Earth in the last lecture, but this concept is so important that it's worth re-deriving in a different way. Let's look at a car going around a track at a steady speed *v*. For convenience, we'll assume the distance *v* traveled from *A* to *B* around the circle in one second is a tiny fraction of the circle.



In one second, a car travels *v* meters at steady speed along a circular road. The velocity vector turns through the same angle as the road, so the two triangles are similar, and $a/v = v/R$, $a = v^2/R$.

(If it isn't, we'll just take a shorter time period—we're interested in the acceleration at an instant of time, so strictly speaking we should take the limit of the two points *A*, *B* on the road closer and closer together.)

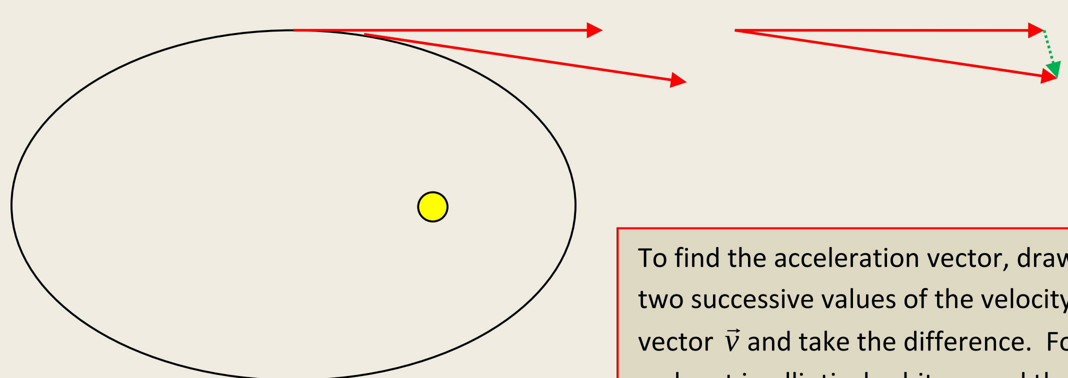
When the line from the car to the center of the circle *C* swings through an small angle, the direction of the velocity vector turns through the same angle, because the direction of the velocity, along the road, is always perpendicular to the radius line form the center of the circle to that point on the road.

The acceleration $\vec{a}$, the change in velocity in one second, is how much has to be added to the first velocity vector $\vec{v}$ to get the second one, that's just the vector from the head of the first velocity vector to the head of the second, when we draw them with their tails together (see diagram).

So the two isosceles triangles *ABC* and *vva* are similar, their sides are in the same ratio, so *a*/*v* = *v*/*R*, and **the acceleration *a* towards the center for motion at a steady speed *v* in a circle of radius *R* is given by:**

$$\boxed{a = \frac{v^2}{R}}$$

What about the acceleration of a moving object where both magnitude *and* direction of velocity are changing? An example is a planet moving in an elliptic orbit: the path curves, but also as the planet gets closer to the Sun, it speeds up. Remember acceleration is a vector! It will have two components: one along the path, given by the rate of increase of speed, one perpendicular to the path, towards the center of curvature (think of the path in that neighborhood as a small part of a circle). The total acceleration is the vector sum of these two. Another way of seeing this is to draw the velocity vectors one second (or some appropriate time unit) apart, and see the acceleration vector as the difference between them.



To find the acceleration vector, draw two successive values of the velocity vector $\vec{v}$ and take the difference. For a planet in elliptical orbit around the Sun, $\vec{a}$ points towards the Sun.

Force

To continue discussing $\vec{F} = m\vec{a}$, we'll now focus on $\vec{F}$, the total force on an object. Long before Galileo, there was a general understanding of contact forces: just pushing something, or pulling it with a rope, say. Galileo made clear that friction and air resistance were also forces, and linked friction with acceleration for a rolling ball, and air resistance with leveling off of acceleration for a fast falling object.

But Newton made the major advances: he realized weight was a force, and he appreciated that forces between objects—any kinds of forces—came in pairs.

"Free Body" Diagrams

In general, several forces act on an object at the same time, and its acceleration is determined by the total force acting. Forces are of course vectors, so must be added like vectors.

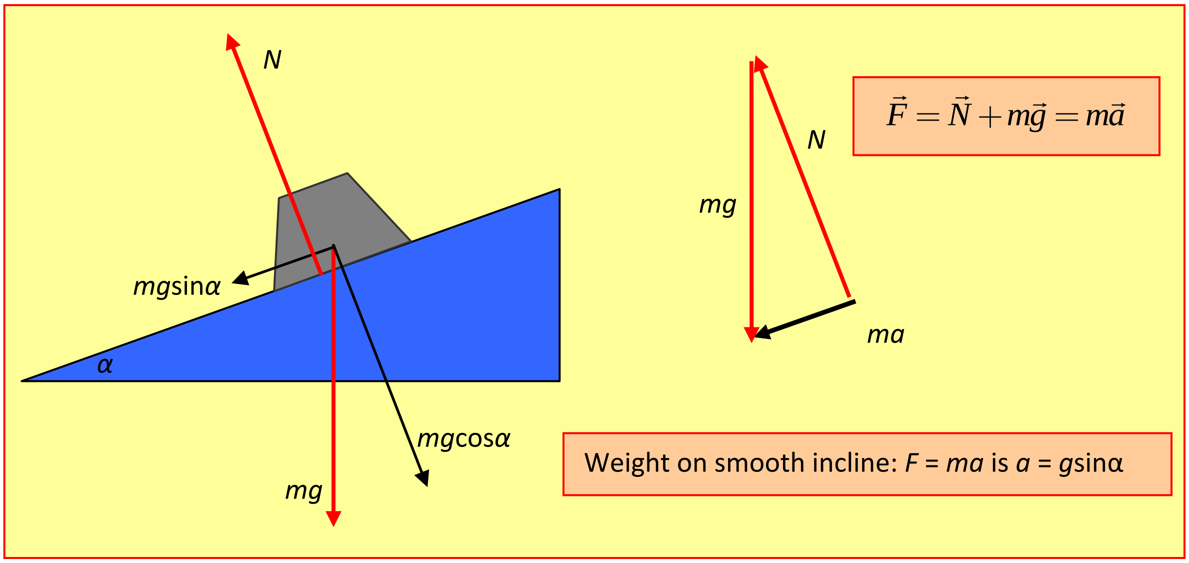
A "free body " diagram is what you get by focusing on one object in a possibly complicated group of interacting things, and drawing the forces acting on that object. This can be tricky—the object is itself acting on others, those forces must be left out.

Also, a free body diagram doesn't imply the object is moving freely—we could draw a free body diagram for someone handcuffed to a lamppost.

The "Normal" Force

If anything pushes against a surface, from Newton's Third Law, the surface pushes back. If it's a smooth, frictionless surface (an idealization, but close to the truth for smooth ice, for example) then the force back is perpendicular to the surface. This is called the "normal" force, where normal has its mathematical meaning of perpendicular to a surface, and is denoted by $\vec{N}$. For a weight resting on a horizontal surface, the normal force is the only force from the surface, even if the surface isn't smooth. In fact, this is a simple example of a free body diagram: the only forces acting are gravity and this normal force.

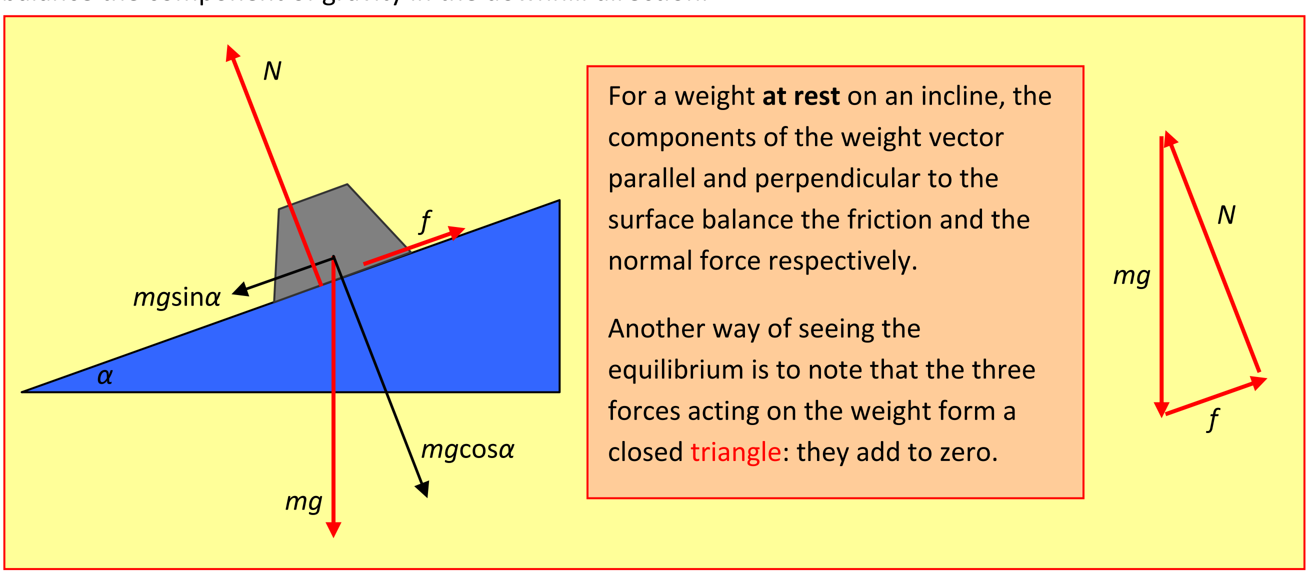
It's not exactly obvious why something as passive seeming as a flat hard surface should be able to exert a considerable force. This is best understood by thinking first of lowering the weight gently on to a spring. The weight compresses the spring, which exerts a greater and greater force back on the weight as it is further compressed, until a point is reached where the spring's force equals the weight. The tabletop or other hard surface behaves in just the same way—the difference being that the tabletop is like a spring that's very hard to compress, so the weight only moves it downwards a very short distance before the resistive force is sufficient to balance the weight. This sagging of the tabletop under a weight is quite easily detected by reflecting a laser beam off the tabletop, and observing the spot to move a little when the weight is placed on the table.



If the weight is on a frictionless sloping surface, it will slide downhill. The two forces in the free body diagram are again weight and the normal force, but they cannot cancel because the normal force, always perpendicular to the surface, is no longer parallel to the weight force. The way to analyze this situation is to resolve the two vectors into components parallel and perpendicular to the surface. The component of weight perpendicular to the surface must cancel the normal force, since acceleration in that direction is blocked, and the acceleration down the slope is given by the component of weight acting in that direction.

Friction

Real surfaces in contact are not frictionless. There is a resistance to motion of one surface sliding on the other. If the surfaces are not moving, for example a weight at rest on an incline, the force is termed static friction. The frictional force $\vec{f}$ on the weight is tangential to the surface, directed uphill to balance the component of gravity in the downhill direction.



Experimentally, it is found that the maximum magnitude of this frictional force is directly proportional to that of the normal force:

$$f = \mu_S N$$

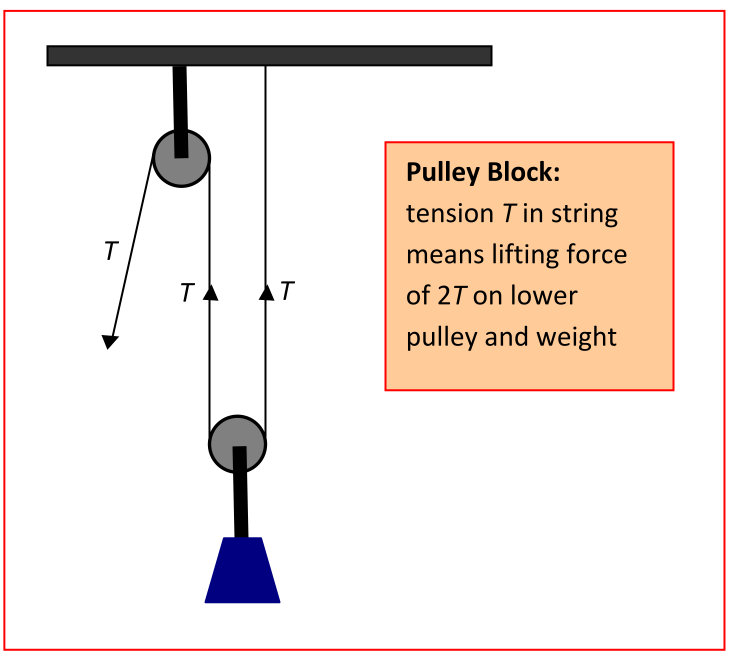
where μ_S is called the coefficient of (static) friction. (Sometimes written $f_S = \mu_S N$.)

Of course, if the surface is horizontal, there is no frictional force unless some other outside force has a horizontal component. Assuming no such forces, then if the angle α is gradually increased from zero, the frictional force will increase. At some α, the weight will begin to slide down: this is the point at which the frictional force reached its maximum value, and $f = \mu_S N$ immediately gives $\mu_S = \tan\alpha$. (See figure).

Once the weight begins to slide, in practice the frictional force decreases somewhat, although it is still proportional to the normal force. A coefficient of kinetic friction is therefore introduced: $f \ (\text{or } f_K) = \mu_K N$.

The frictional force depends on the materials of the two surfaces, but does not depend on the apparent area of contact. It is believed that the true area of contact, at the atomic scale, is far smaller than what we see, and that in fact this true area increases linearly with the normal force.

Tension in a String



A string in tension exerts a force on both ends. These forces are directed along the string, but are in opposite directions: the string pulls inwards on both its ends. Usually strings have negligible mass relative to other objects in the configuration, so the total force on a string must be zero, meaning the tensions at the two ends are equal. The string doesn't have to have ends—a closed loop over two pulleys will work.

And the tension in a single string can be used more than once to lift a load, as in this pulley block.

Newton's Third Law

Newton's Third Law is the best known and least understood of his laws. Everybody can quote "action equals reaction" but few realize **these forces always act on different bodies.** Therefore **the action and reaction can never both appear in a free body diagram.** For a person standing on the floor, the reaction to the weight force—the Earth pulling the person downwards—is **not** the floor pushing up, it's the person pulling the Earth upwards gravitationally. The action/reaction force pair are forces of the same type, in this case gravitational attraction. The other pair of forces in the picture is the pair of contact forces, the person pushing the floor down, the floor pushing the person up.