Solving Newton’s Second Law Problems

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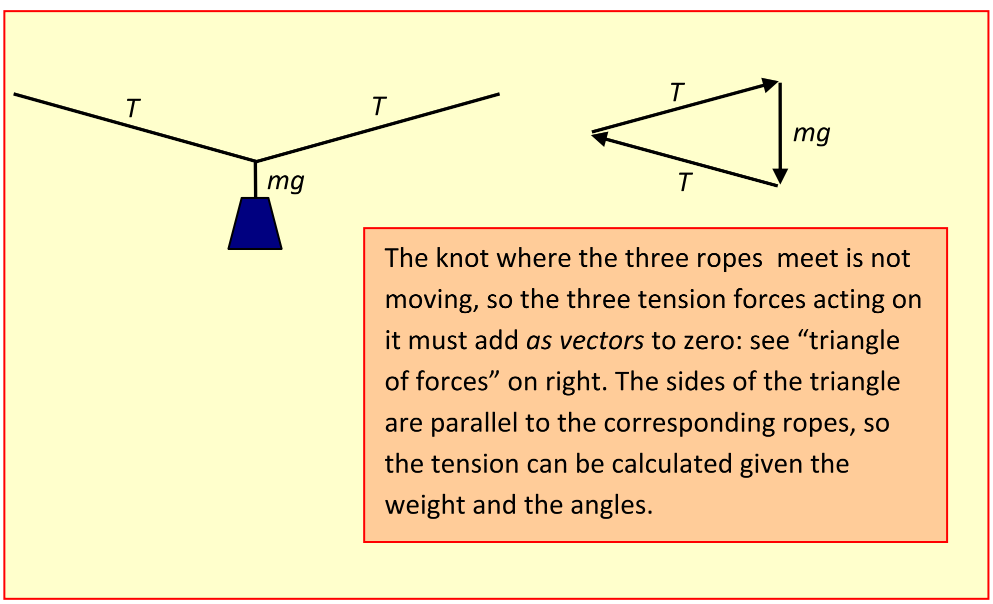
Zero Acceleration Problems: Forces Add to Zero

The Law is $\vec{F} = m\vec{a}$: the acceleration of a given body is given by the net force on that body divided by the mass. But it’s often surprisingly difficult to find the force on a particular body. The strategy is to mentally isolate that body, and enumerate the forces acting on it—this is the “free body” diagram. These vector forces must then be added correctly—head to tail—and the resulting total force found. This will give the direction of the body’s acceleration.

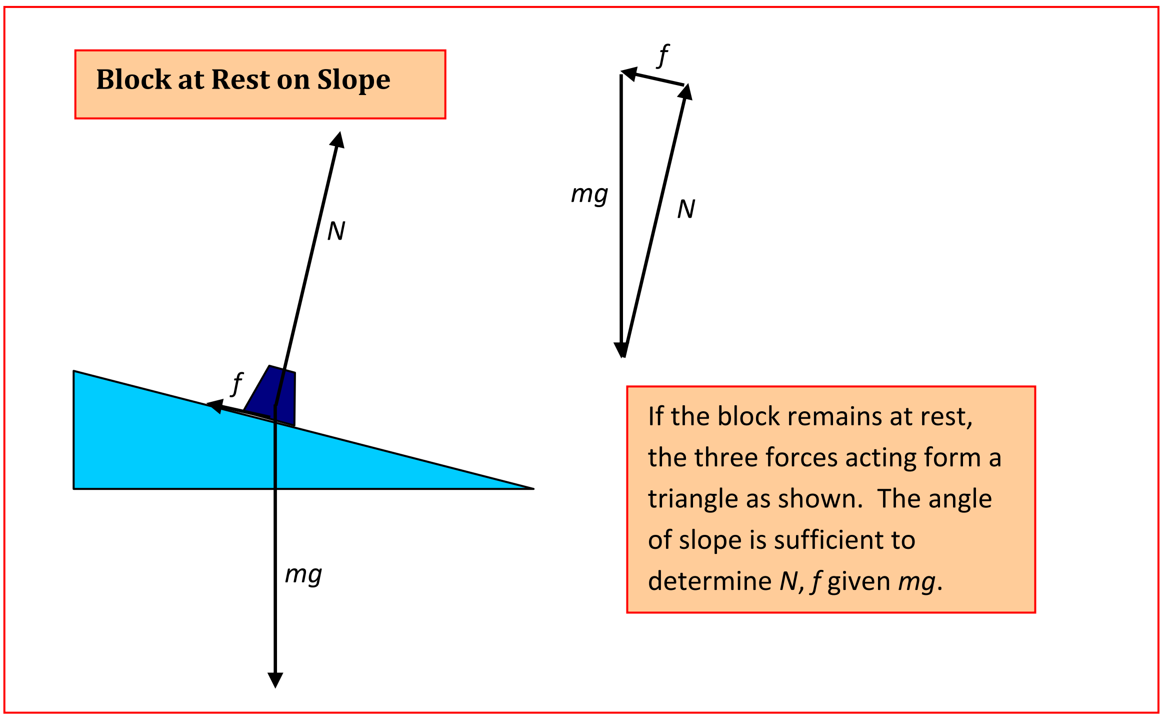
We’ll begin with the static situation: no acceleration. For example, we can find the tension in the ropes below by noticing that the total force on the know where the ropes come together must be zero, since the know isn’t moving. We draw a “triangle of forces” , the sides parallel to the corresponding ropes. The angles in this triangle are therefore determined by the angle of slope of the ropes, so with simple trig the ratio of *T* to *mg* an be found. **Notice that the force vectors must all point the same way going around the triangle.** Otherwise, they won’t add to zero. This also works for side ropes at different angles, it’s just more complicated.



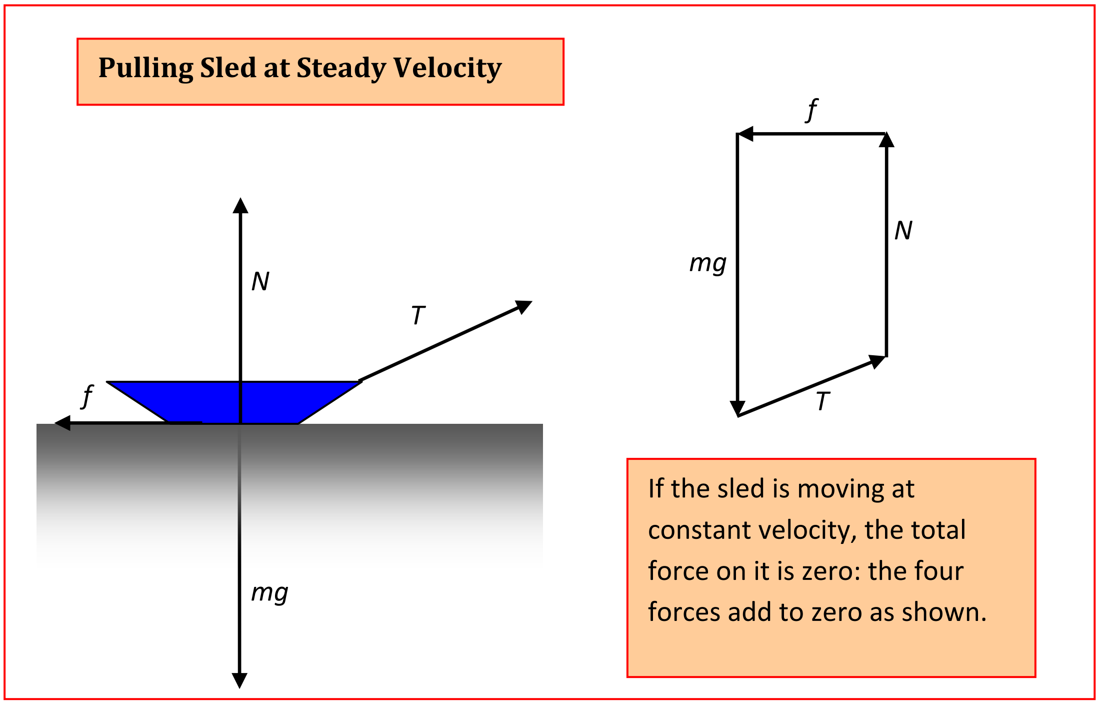
The knot where the three ropes meet is not moving, so the three tension forces acting on it must add *as vectors* to zero: see “triangle of forces” on right. The sides of the triangle are parallel to the corresponding ropes, so the tension can be calculated given the weight and the angles.

For a block at rest on a slope, the upward force of the slope's surface on the block must exactly counterbalance the block's weight *mg*. This force from the slope has two components: the normal force *N*, meaning the force perpendicular to the surface resulting from the springiness of the surface, the weight has compressed it slightly and it's pushing back, and the frictional force *f* preventing the weight from sliding down the slope.

From the triangle of forces (all going round the same way, and therefore adding to zero!) one can find *N*, *f* in terms of *mg,* since the angles of the triangle are determined by the slope.



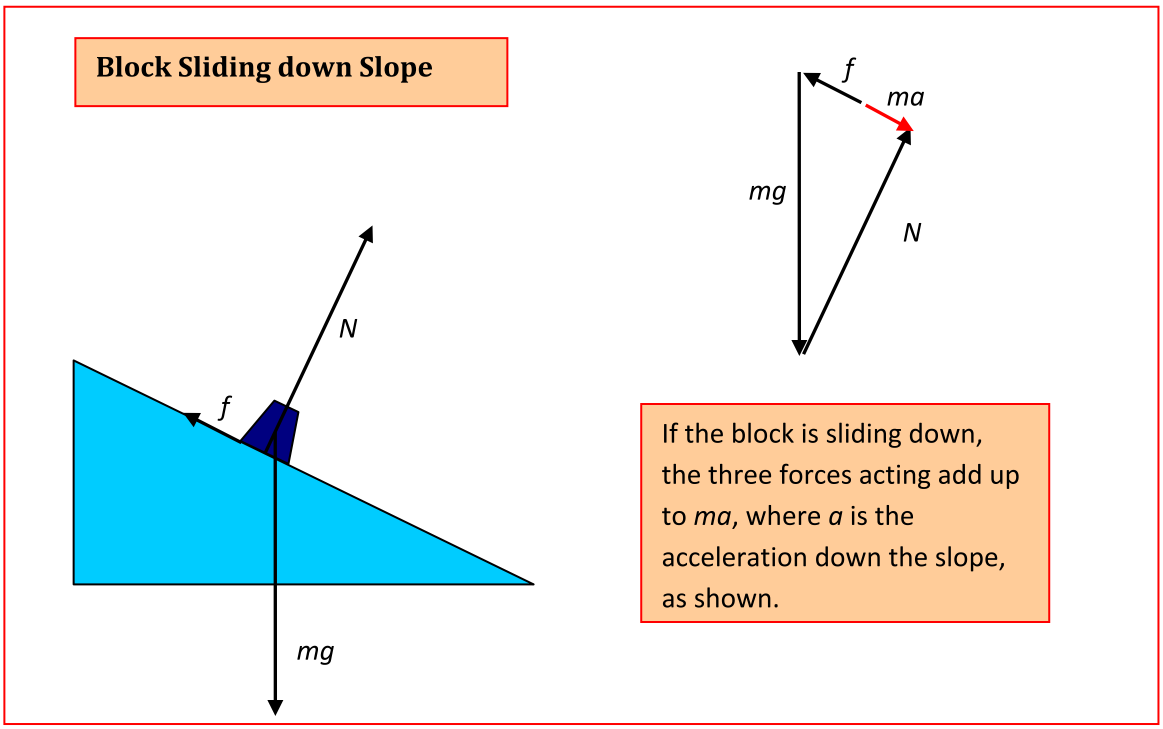
A steady velocity situation is of course the same as a static one: there's no acceleration, so the vector total of forces acting on the body must be zero. Take the case of a sled



being pulled using a rope at a fixed angle as in this diagram. Separating the force from the ground into normal and frictional components, we have a quadrilateral of forces. Since the sled is moving, $f = \mu_K N$ (the coefficient of kinetic or sliding friction). This means that given the angle of the rope, the tension needed to maintain speed can be found. (Write out the equations for vertical and horizontal components.)

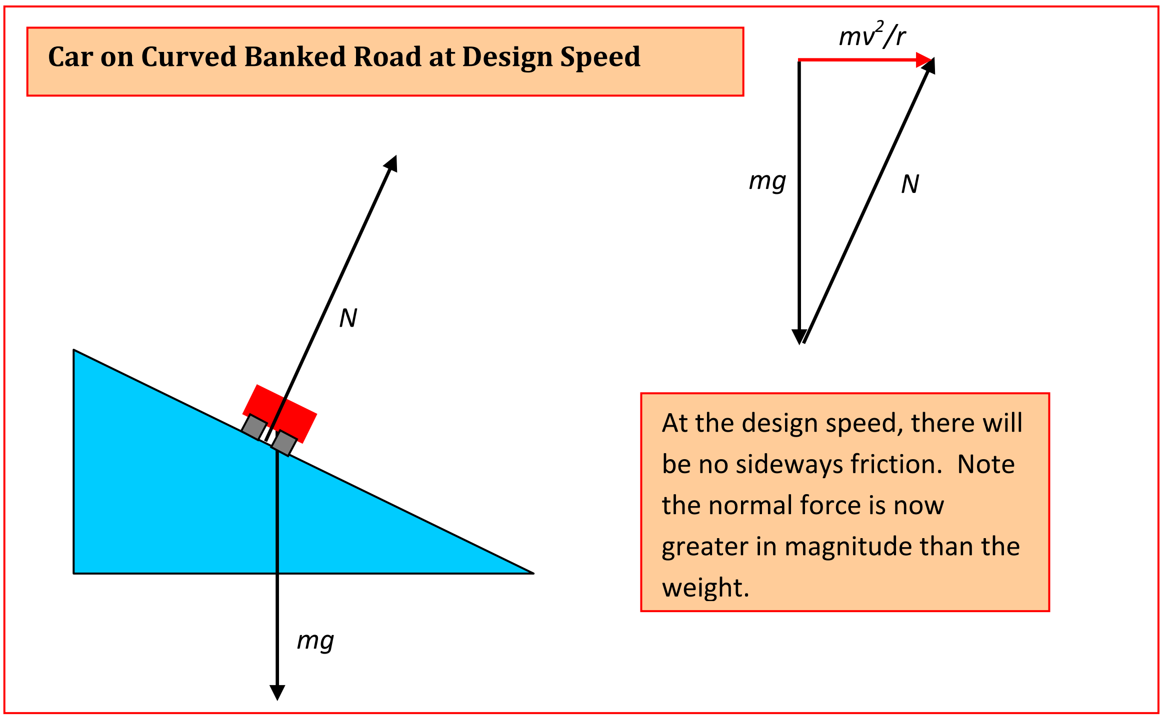
Object Accelerating: Forces Add to Mass × Acceleration

Consider now a block accelerating as it slides down a slope, with frictional resistance $f = \mu_K N$. The acceleration vector is directly down the slope, the frictional force directly up the slope:.



The three force vectors no longer add to zero, they add to $m\vec{a}$. To solve this problem, equate the force components parallel to the slope, then those perpendicular to the slope.

If a road going around a bend is suitably banked, the component of the normal force pointing towards the center of the curve will contribute to the necessary v^2/r acceleration. If the road is not banked, the centripital acceleration is generated entirely by sideways friction, and especially on wet roads this can be problematic.

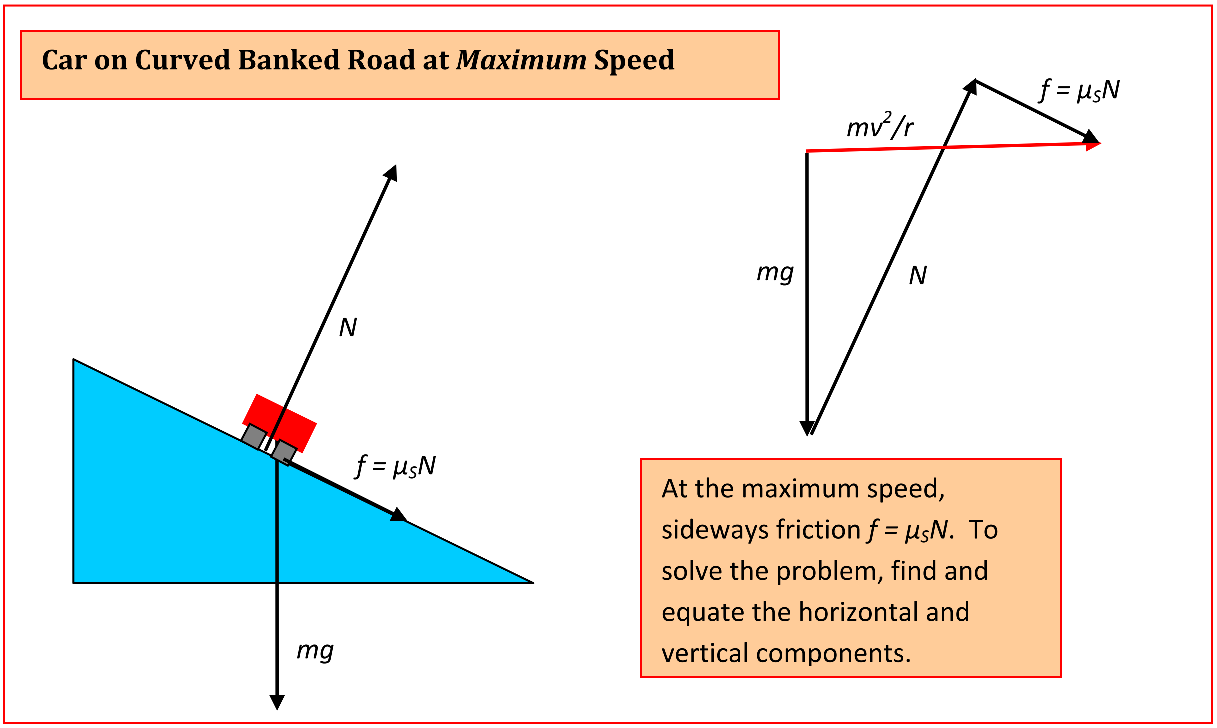


Car on Curved Banked Road at Design Speed

However, for a given angle of banking, equating mv^2/r to the horizontal component of the normal force determines v: so it only works perfectly for one speed, the "design speed' of that stretch of road, and at that speed you could make it round the curve on a sheet of ice. But for speeds anywhere near the design speed, less frictional force is required on a banked road.

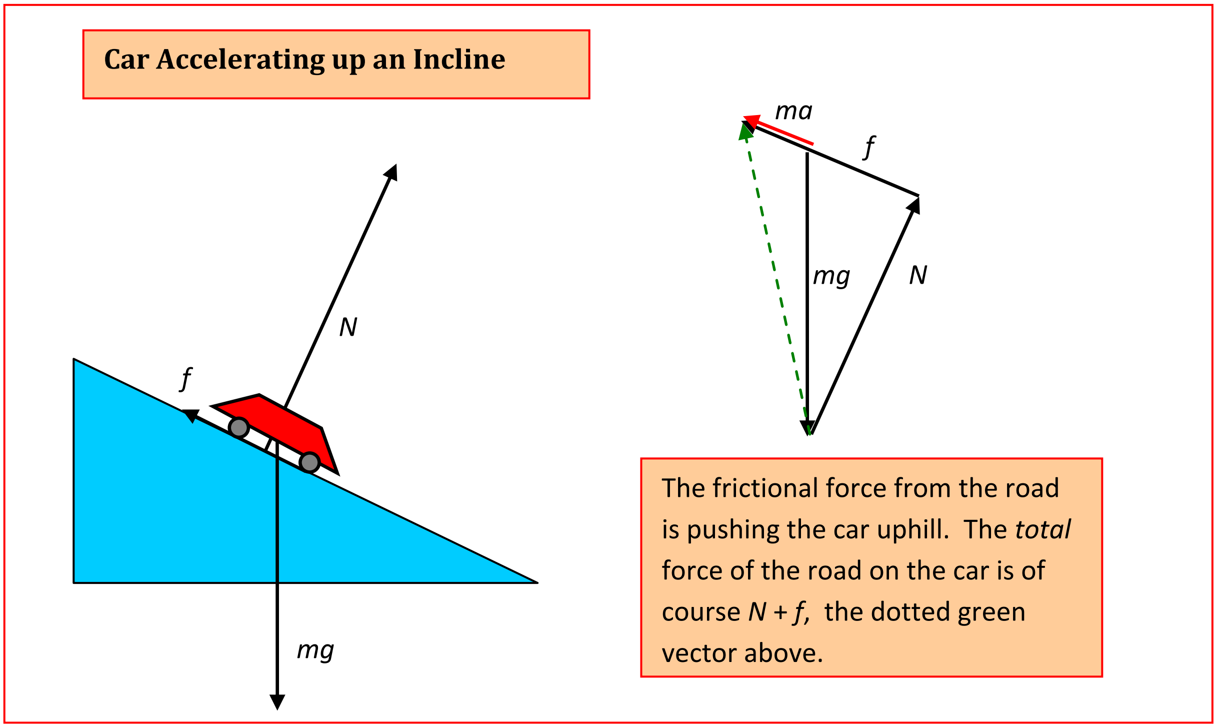
For a given coefficient of (static) friction, there is a maximum speed before the car deviates from the circular path.

Here is the diagram for $\vec{F} = m\vec{a}$:



Actually, if $\mu_S = 1$ (realistic for some tires) and the road is banked at 45 degrees, the maximum safe speed, according to this analysis, increases without limit. (Try drawing the vectors!) However, we've ignored the possibility that the car might begin to roll up the banking.

For a car accelerating uphill, the external force in $\vec{F} = m\vec{a}$ causing the acceleration is the frictional force acting uphill, minus the component along the slope of the weight.



The dashed vector is the total force from the surface on the car, this vector plus $m\vec{g}$ gives $m\vec{a}$. The force from the seat on the driver would be in exactly the direction of the dashed vector.

Problem Solving Strategy

It's all about solving $\vec{F} = m\vec{a}$ for a particular body. The first thing is to figure out which body, and see what you can say about its acceleration: you might not know its magnitude, but you probably know its direction. If it's going in a circle at constant speed, you know it's accelerating towards the center. If it's going in a straight line, the acceleration must be along that line. (Of course, there are trickier cases: a car picking up speed as it goes around a bend, a planet in an elliptical orbit, etc.)

Now you enumerate the forces on the body, and use $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a}$. By this I mean draw a diagram with the forces drawn as vectors, with the directions correct (you know normal forces are always perpendicular to the surface, friction along the surface) and try to represent this equation, taking for example three forces, as a head-to-tail vector sum equaling the $m\vec{a}$ vector, as in the examples shown above. Don't represent the vectors in terms of their components until you can see how they add up: this will give you a clearer picture of what's going on, without the clutter. Only then, to do actual calculations, put the vectors in terms of their components in two directions at right angles, and write, say, $F_x = ma_x$, etc.