

Electric Potential

Physics 2415 Lecture 6

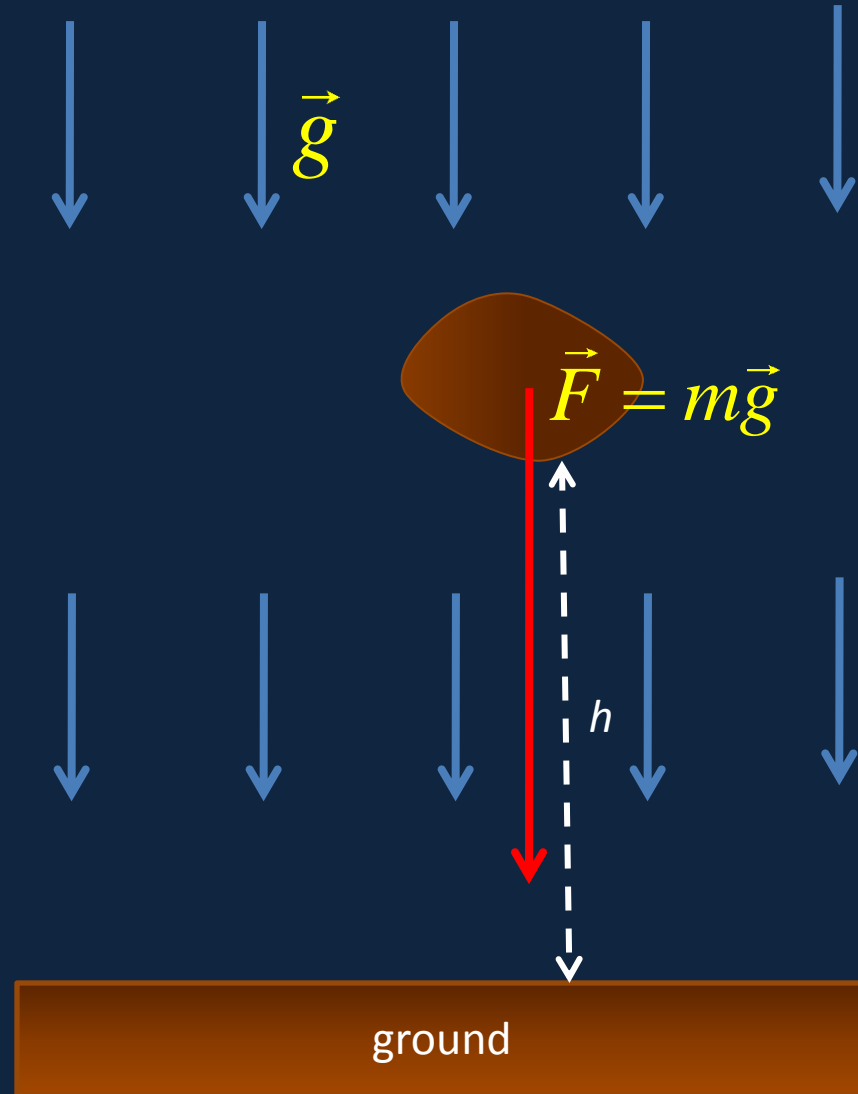
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Today's Topics

- Some reminders about gravity: mgh and its electric cousin
- Inverse square law and its potential
- Field lines and equipotentials

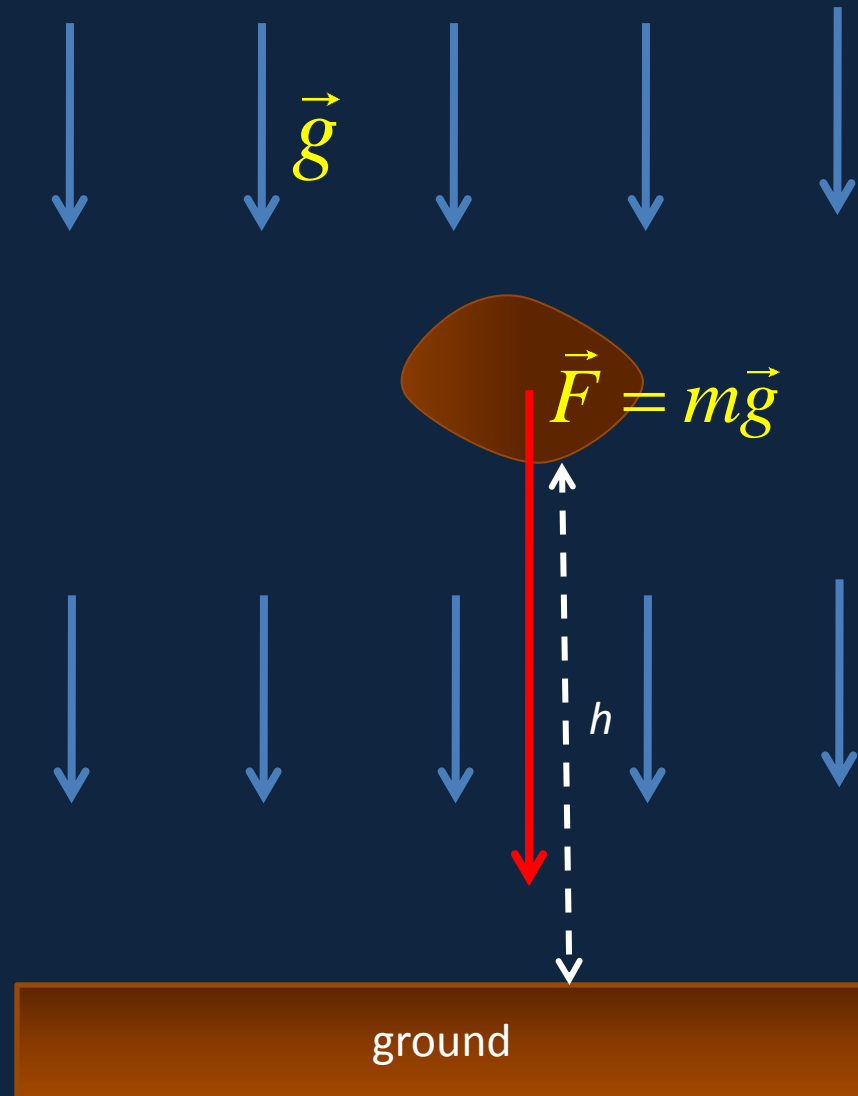
Lifting a Rock

- Near the Earth's surface, the gravitational field vector points vertically down, and has constant magnitude g , the force on a mass m is $\vec{F} = m\vec{g}$.
- The work done in lifting mass m through height h is mgh : this is the **potential energy**.



Lifting a Rock

- The work done in lifting mass m through height h is mgh : this is the **potential energy**—defined to be **zero at ground level**, but could take some other level as zero, only *differences* of potential energy matter.
- The PE per unit mass, gh , is called the **(gravitational) potential**.

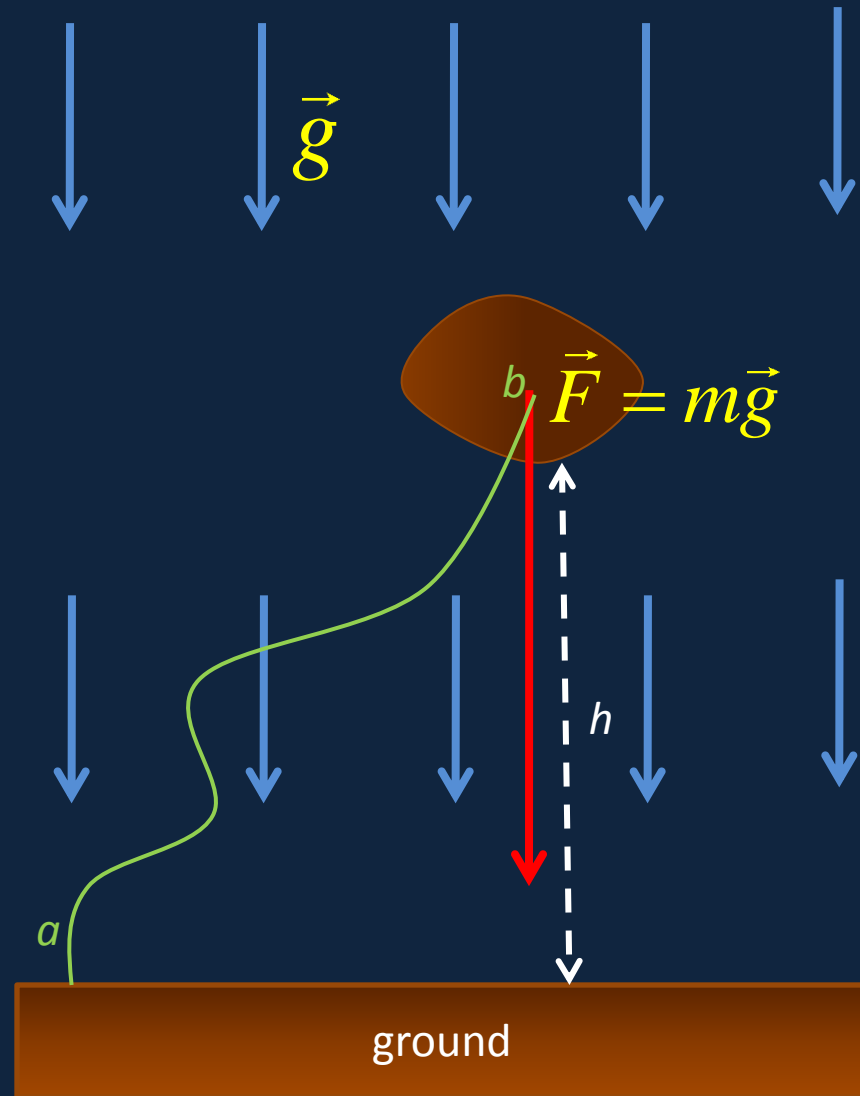


Lifting a Rock along a Wavy Path

- Suppose we lift up the heavy rock erratically, following the wavy green path shown. Our work against gravity only involves the component of the gravitational force pointing along the path:

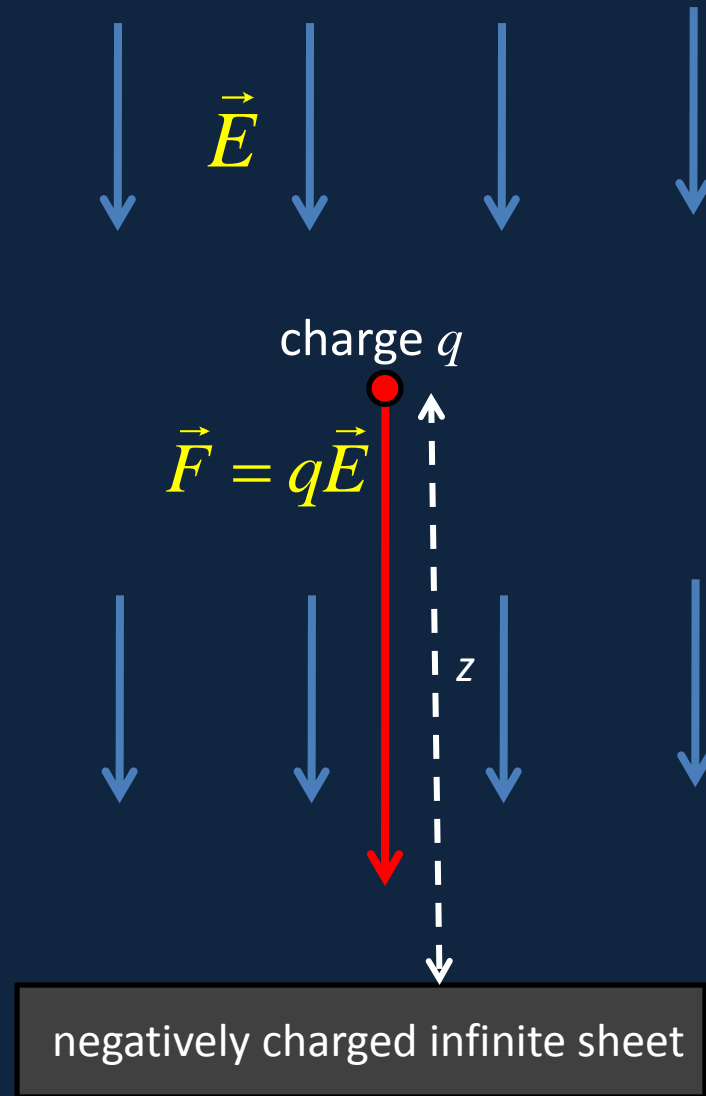
$$W = -\int_a^b m\vec{g} \cdot d\vec{\ell}$$

- Or, equally, only the upward component of $d\vec{\ell}$ counts, and $W = mgh$.



Electric Potential of a Negative Sheet

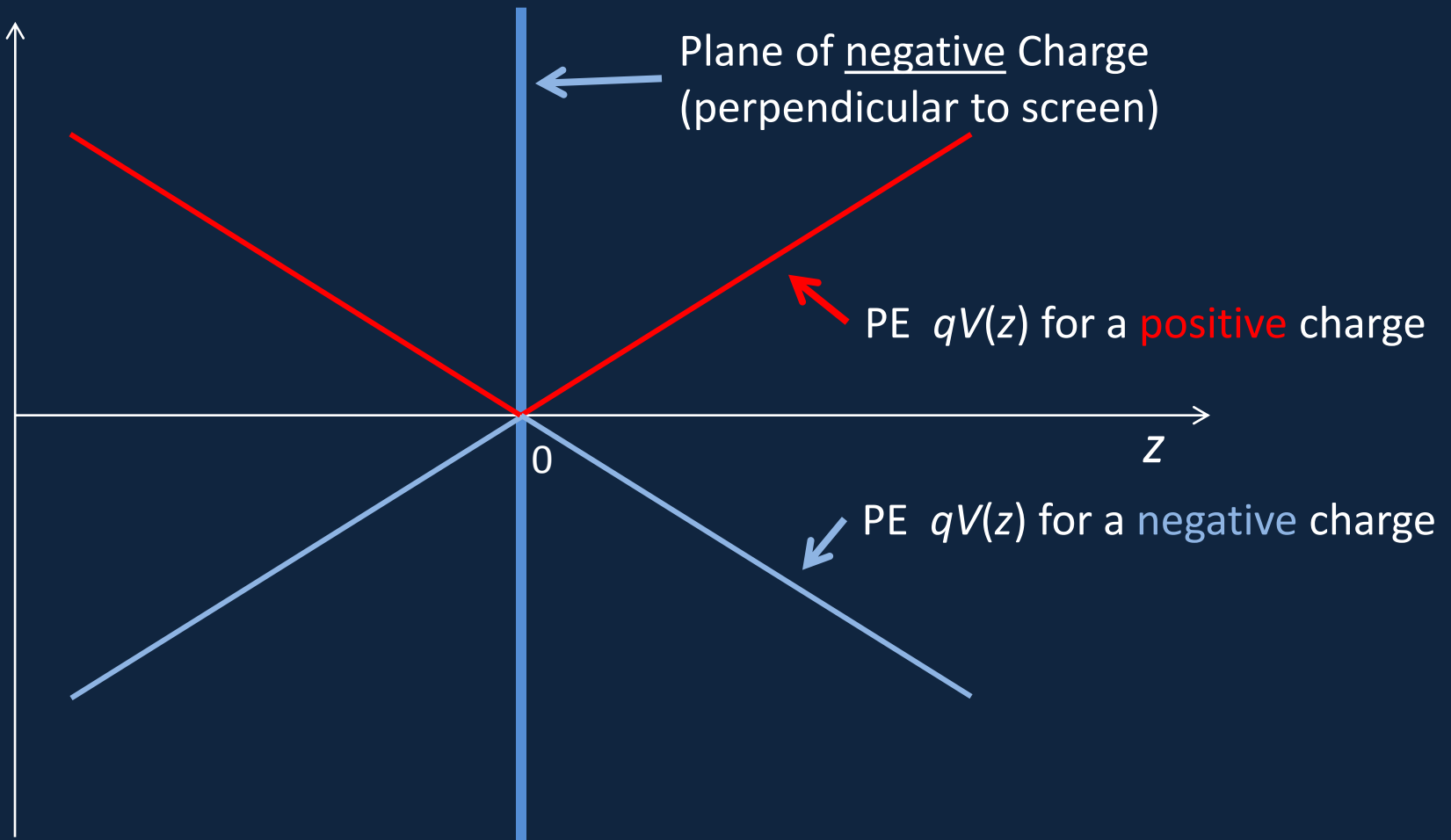
- Imagine an infinite sheet of negative charge, σ C/m².
- On either side of the sheet there is a **uniform electric field**, strength $E = \sigma / 2\epsilon_0$, directed towards the sheet.
- To move a + charge q from the sheet distance z takes work qEz .
- The electric potential difference $V(z) - V(0) = Ez = \sigma z / 2\epsilon_0$ and **this $\times q$ becomes KE if the charge is “dropped” to the sheet.**



Potential, Potential Difference and Work

- We've seen that the electric field of a uniform infinite sheet of negative charge is constant, like the Earth's gravitational field near its surface.
- Just as a gravitational **potential difference** can be defined as **work** needed **per unit mass** to move from one place to another, electric potential difference is **work** needed **per unit charge** to go from a to b , say.
- The **standard unit** is: 1 volt = 1 joule/coulomb

Potential Energy of a Charge Near an Infinite Plane of Negative Charge

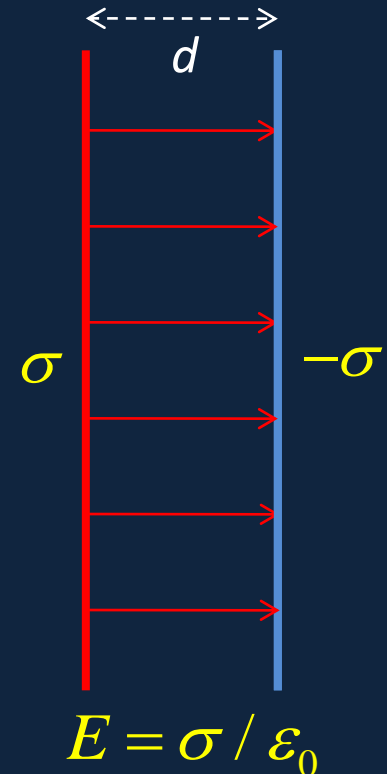


Electric Field and Potential between Two Plates Having Opposite Charge

- Separation d is small compared with the size of the plates, which carry uniform charge densities $\pm\sigma$.
- The electric force on a unit charge between the plates $E = \sigma / \epsilon_0$ N/Coul.
- The voltage (potential difference) between the plates is the work needed to take unit charge from one to the other,

$$V = Ed$$

- Note from this that E can be expressed in volts/meter.



Units for Electric Potential and Field

- Potential is measured in volts, to raise the potential of a one coulomb charge by one volt takes one joule of work:
- One volt = one joule per coulomb
- An electric field exerts a force on a charge, measured in newtons per coulomb.
- Since one joule = one newton x one meter, electric field is equivalently measured in volts per meter.

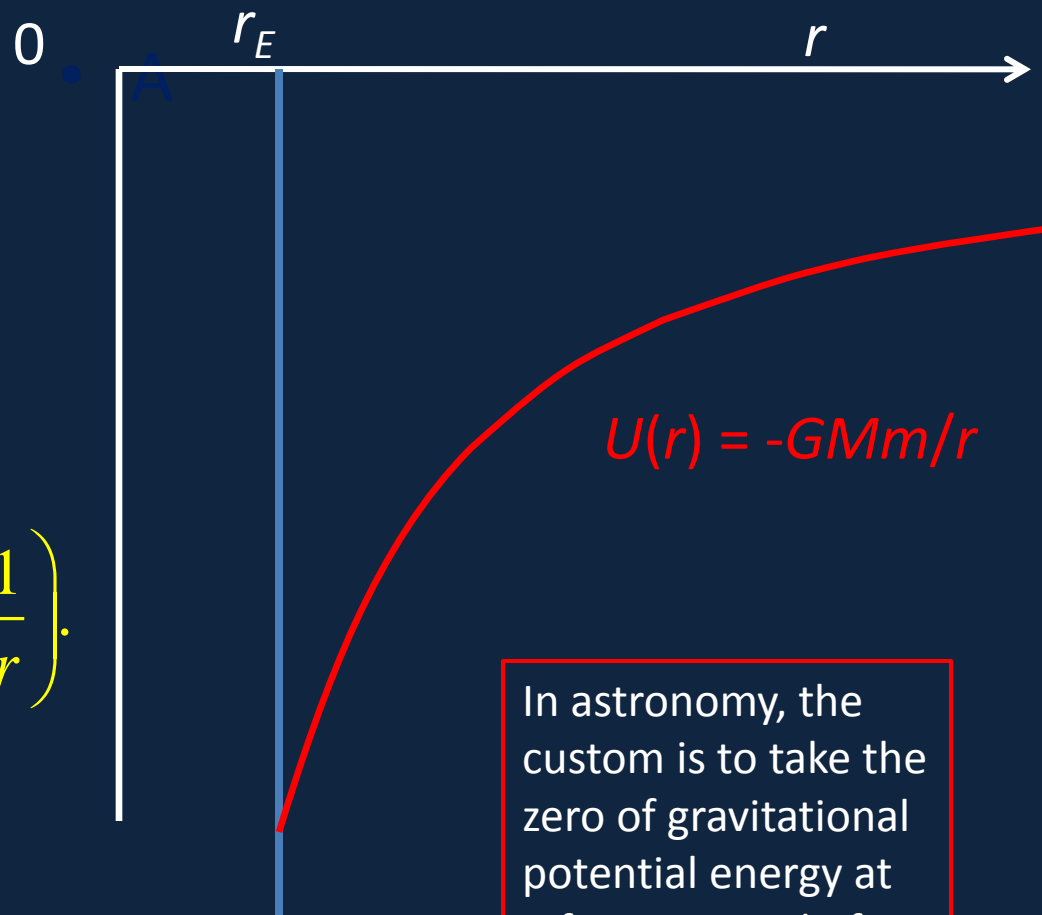
Gravitational Potential Energy...

- ...on a bigger scale!
- For a mass m lifted to a point r from the Earth's center, far above the Earth's surface, the work done to lift it is

$$W = \int_{r_E}^r \frac{GMm}{r^2} dr = GMm \left(\frac{1}{r_E} - \frac{1}{r} \right).$$

- If $r = r_E + h$, with h small,

$$W = GMm \frac{r - r_E}{rr_E} \approx \frac{GMmh}{r_E^2} = mgh.$$



Electric Potential Outside a Uniformly Charged Spherical Shell

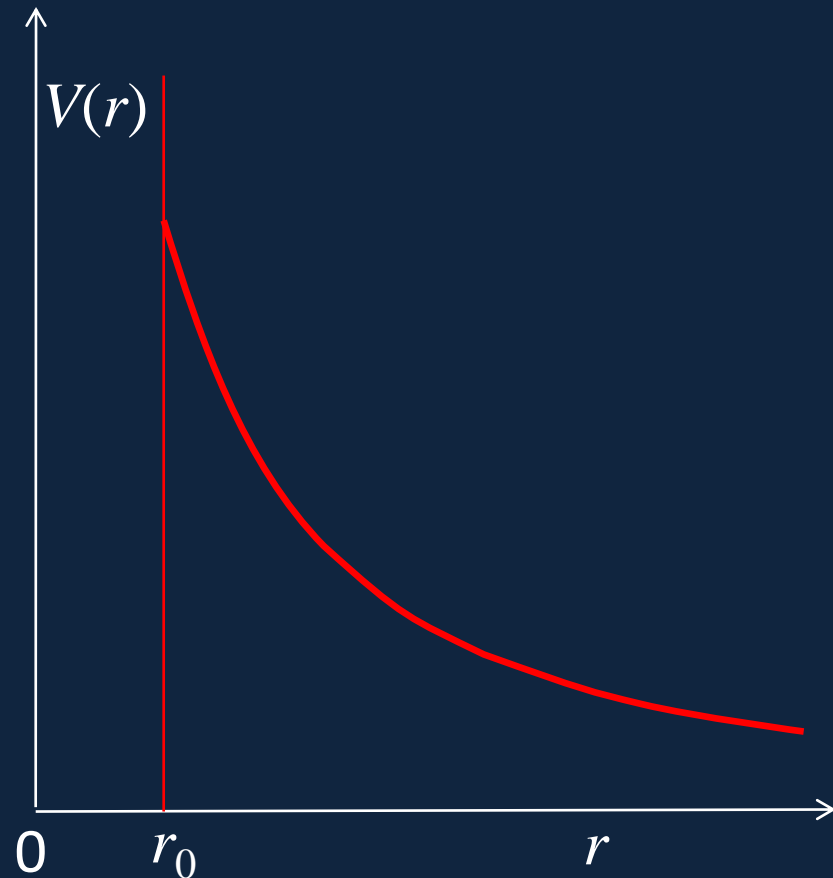
- Recall the electric field is

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q\hat{r}}{r^2}$$

precisely the same form as in gravitation—except this points *outwards*!

- Therefore the PE **must also have the same form**—taking it zero at infinity,

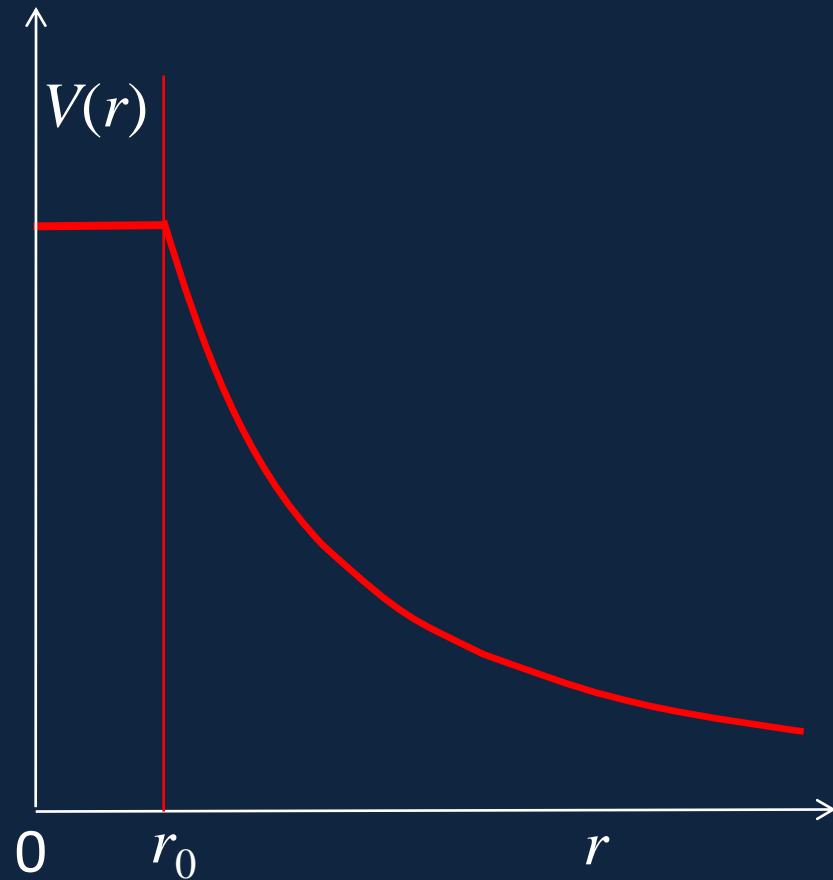
$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$



Electric Potential *Inside* a Uniformly Charged Spherical Shell

- The electric **field** *inside* a spherical shell is **zero** everywhere—so it takes **zero work** to move a charge around. The gravity analog is a flat surface: the **potential is constant**—but *not* zero, equal to its value at the surface:

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_0} \quad \text{for } r \leq r_0$$



Potential Outside *any* Spherically Symmetric Charge Distribution

- We've shown that for a uniform spherical shell of charge the field outside is

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

- **Any** spherically symmetric charge distribution can be built of shells, so this formula is true **outside** any such distribution, with Q now the **total charge**.
- It's **true** even **for a point charge**, which can be regarded as a tiny sphere.

Potential Energy Hill to Ionize Hydrogen

- The proton has charge $+1.6 \times 10^{-19} \text{C}$, giving rise to a potential

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} = 9 \times 10^9 \times \frac{1.6 \times 10^{-19}}{r}$$

- Taking the Bohr model for the ground state of the H atom, the electron circles at a radius of $0.53 \times 10^{-10} \text{m}$, at which $V(r) = 27.2 \text{ V}$.
- The natural energy unit here is the **electron volt**: the work needed to take one electron from rest up a one volt hill. But in H the electron already has $\text{KE} = 13.6 \text{eV}$, so only another 13.6eV is needed for escape.

Potential Energy Hill for Nuclear Fusion

- If two deuterium nuclei are brought close enough, the attractive nuclear force snaps them together with a big release of energy.
- This **could solve the energy problem**—but it's hard to get them close enough, meaning about 10^{-15} m apart.
- Each nucleus carries positive charge e , so

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \approx 9 \times 10^9 \times \frac{1.6 \times 10^{-19}}{2 \times 10^{-15}} \approx 10^6 \text{ eV}$$

- This is the problem with fusion energy...

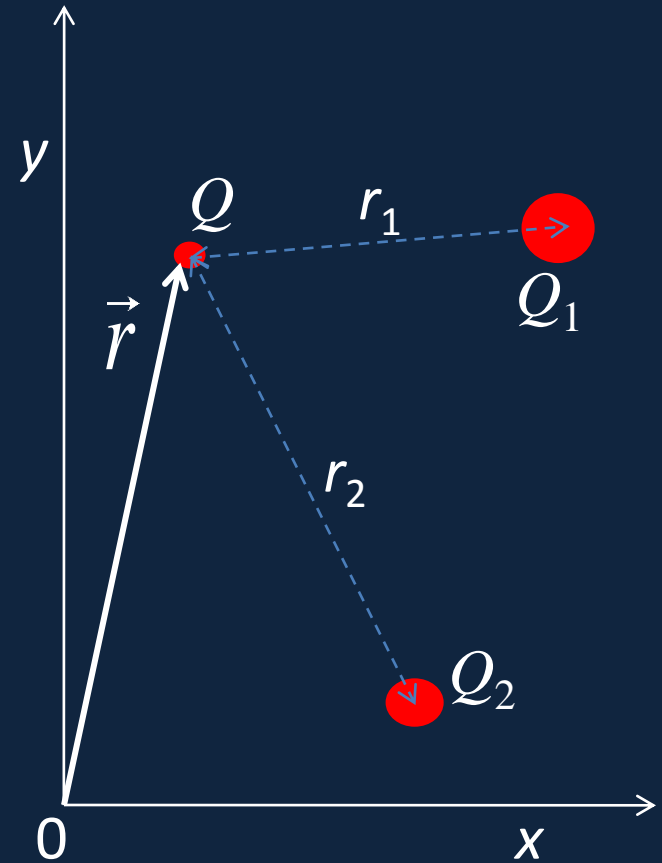
Potential Energies Just Add

- Suppose you want to bring one charge Q close to two other fixed charges: Q_1 and Q_2 .
- The electric field Q feels is the sum of the two fields from Q_1 , Q_2 , the work done in moving $d\vec{\ell}$ is

$$\vec{E} \cdot d\vec{\ell} = \vec{E}_1 \cdot d\vec{\ell} + \vec{E}_2 \cdot d\vec{\ell}$$

so since the potential energy change along a path is work done,

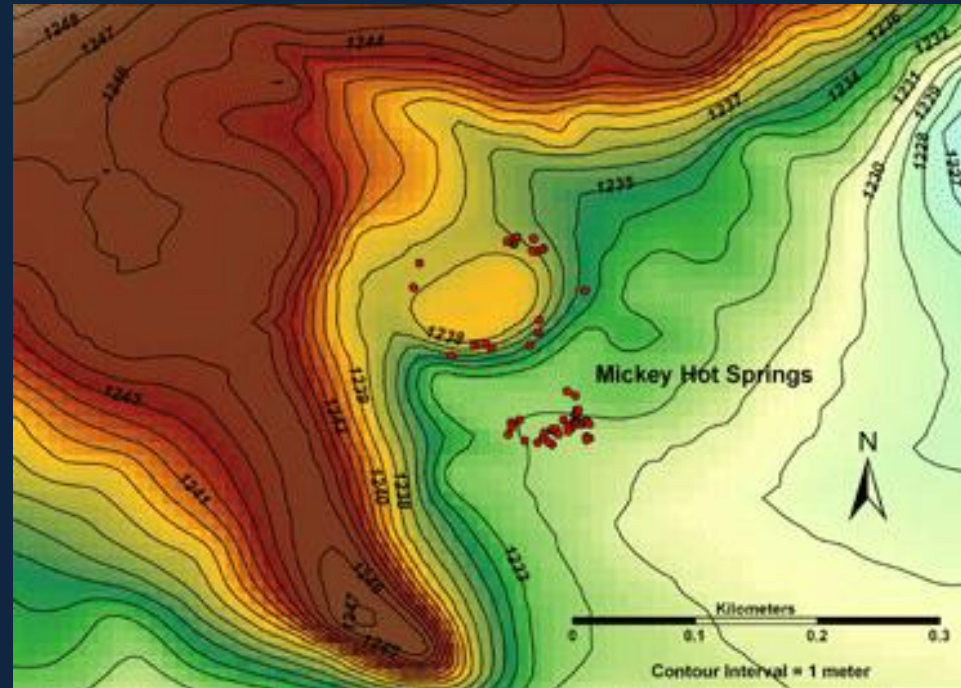
$$V(\vec{r}) = V_1(\vec{r}) + V_2(\vec{r})$$



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right)$$

Equipotentials

- Gravitational equipotentials are just contour lines: lines connecting points (x,y) at the same height. (Remember $PE = mgh.$)
- It takes no work against gravity to move along a contour line.
- *Question:* What is the significance of contour lines crowding together?



Electric Equipotentials: Point Charge

- The potential from a point charge Q is

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

- Obviously, **equipotentials are surfaces** of constant r : that is, spheres centered at the charge.
- In fact, this is also true for gravitation—the map contour lines represent where these spheres meet the Earth's surface.

Plotting Equipotentials

- Equipotentials are surfaces in three dimensional space—we can't draw them very well. We have to settle for a two dimensional slice.
- Check out the representations [here](#).

