DC Circuits II

Physics 2415 Lecture 13

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Today's Topics

- Kirchhoff's rules Review
- RC Circuits
- Ammeters and Voltmeters

General Circuits: Kirchhoff's Rules

- **Junction Rule:** when several wires meet at a point, the total current flowing into the point must equal the total current flowing out. Charge cannot disappear, or pile up at a point.
- **Loop Rule:** the total potential (voltage) change on following wires around a loop to your starting point must be zero.

- (*The loop rule is equivalent to saying that if you follow some random path on a hillside, and get back eventually to your starting point, your net change in height above sea level is zero*.)

Using Kirchhoff's Rules

- **Junction Rule:** as you draw the circuit diagram, use this rule as much as you can to get the smallest number of different unknown currents (one current for each loop).
- And, the number can sometimes be reduced by symmetry.
- **Loop Rule:** the total potential (voltage) change on following wires around a loop to your starting point must be zero: that's adding voltages from batteries to the right IR for each resistance.

Clicker Question

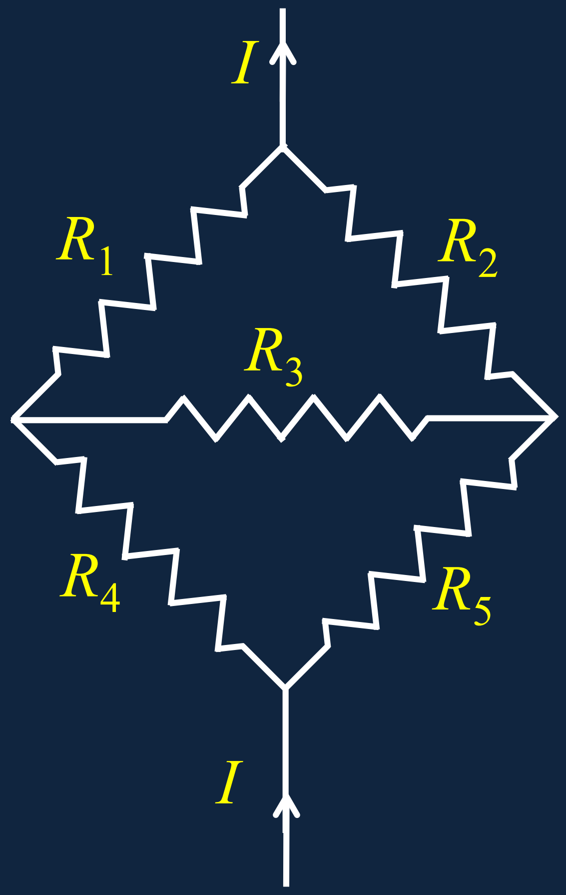
- A current I flows through the network of resistances (all different) shown.
- What is the minimum number of unknown currents it is necessary to introduce to find the total effective resistance using Kirchhoff's laws?

A. 2

B. 3

C. 4

D. 5



Clicker Answer

- A current I flows through the network of resistances (all different) shown.
- What is the miminum number of unknown currents it is necessary to introduce to find the total effective resistance using Kirchhoff's laws?

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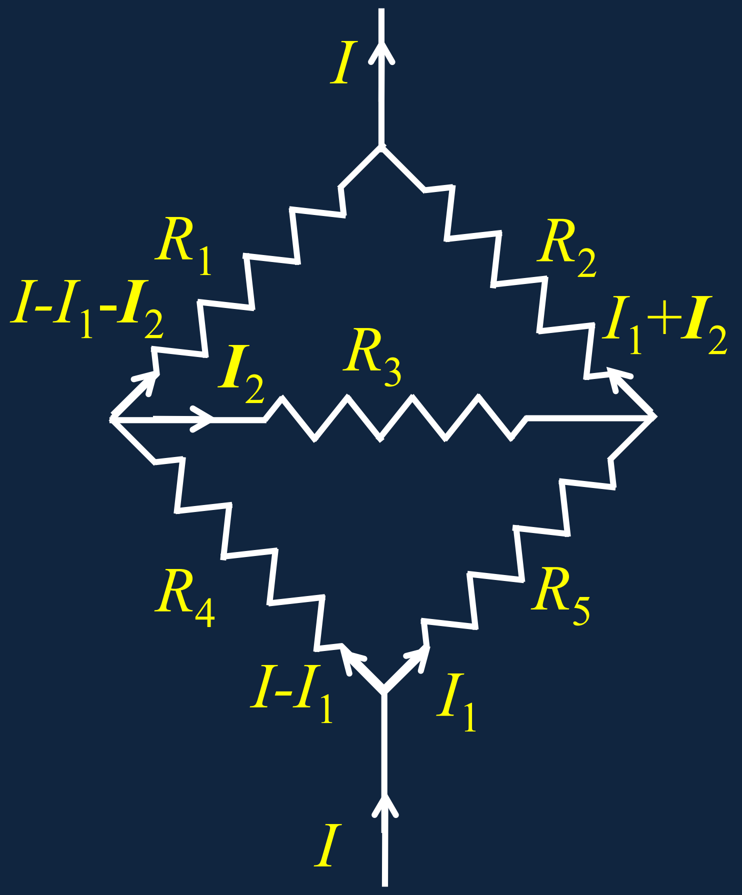


The Loop Equations

- Going round both loops clockwise,

$$\left(I-I_1\right)R_4+I_2R_3-I_1R_5=0$$

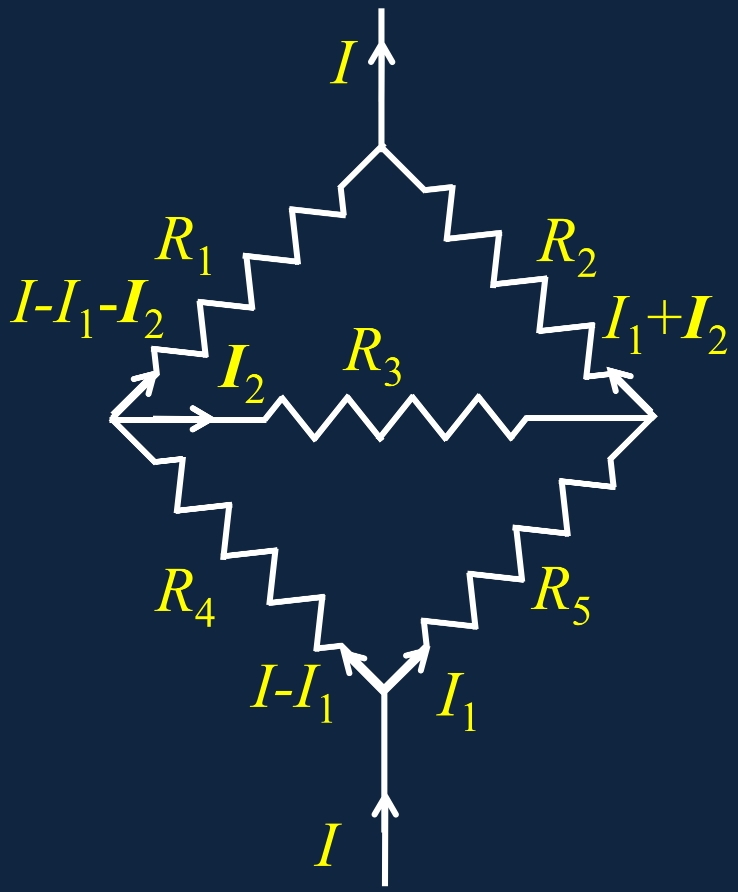
$$\left(I-I_2\right)R_1-\left(I_1+I_2\right)R_2-I_2R_3=0$$

and rearranging:

$$\left(R_4+R_5\right)I_1-R_3I_2=R_4I$$

$$R_2I_1+\left(R_1+R_2+R_3\right)I_2=R_1I$$

Set I = 1 and solve for I_1, I_2.



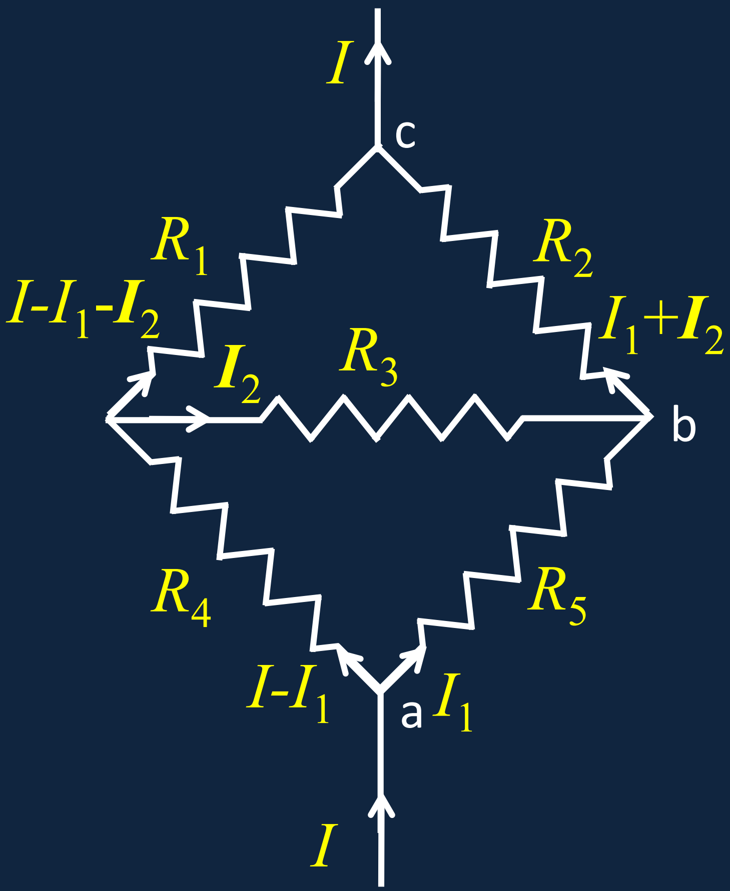
The Loop Equations

- Having found I_1, I_2 we can find the voltage drop: along the path abc (see figure) V drops by

$$V = I_1 R_5 + \left(I_1 + I_2 \right) R_2$$

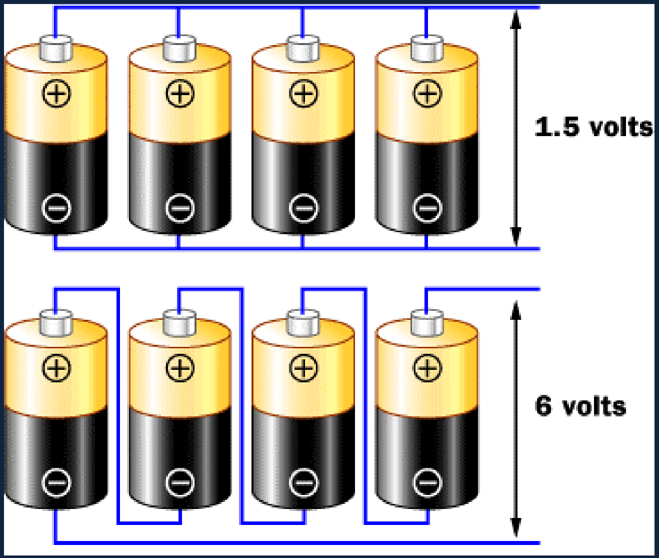
and from $V = IR$ we can find the resistance R of this network.

- If there are batteries within a network, their emf must obviously be included in equating the total voltage change on going around a loop to zero—be careful about the battery direction!



Batteries in Series and Parallel

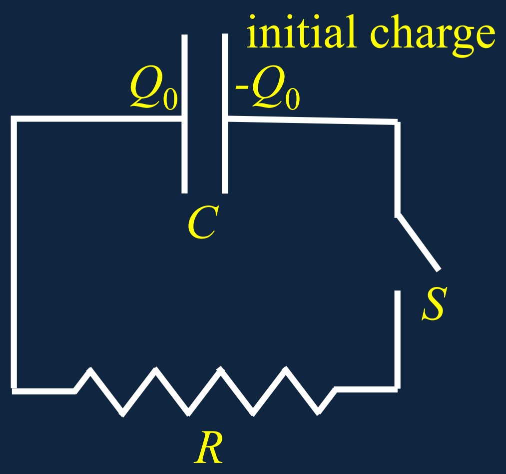
- For batteries in series, the voltages add, in parallel, for identical batteries, they’re equivalent to a larger battery of the same voltage.
- If you put batteries of different voltages in parallel, the stronger will charge the weaker.



RC Circuits

Video!

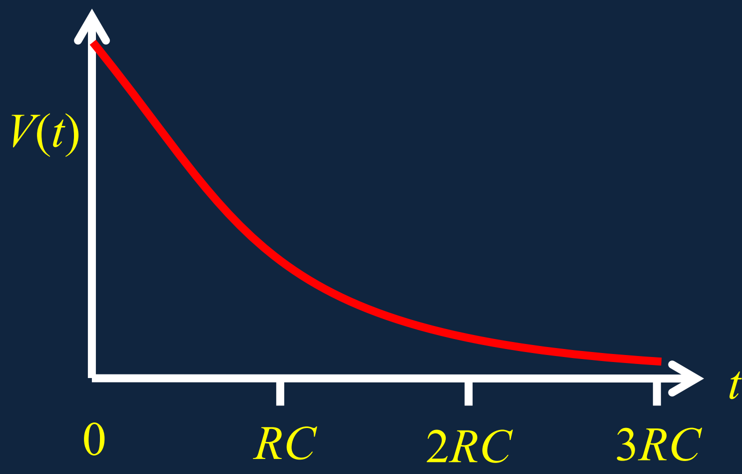
- Closing switch S connects the plates of a charged capacitor by a resistance R. How fast does the charge go down?



$$V(t) = \frac{Q(t)}{C} = RI(t) = -R\frac{dQ(t)}{dt}$$

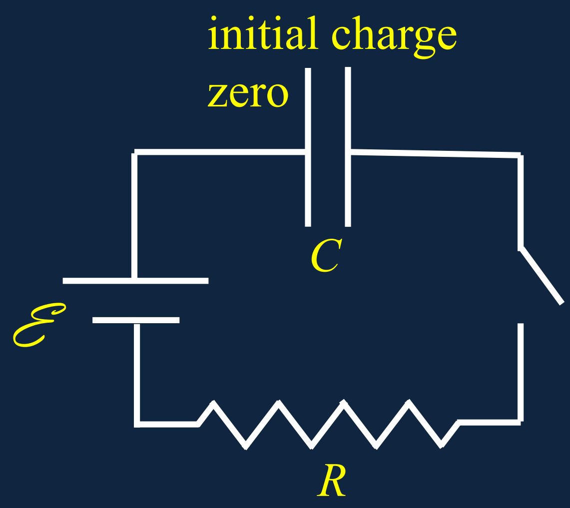
$$\frac{dQ(t)}{dt} = -\frac{Q(t)}{RC}, \quad Q(t) = Q_0 e^{-t/RC}$$

$\tau = RC$ is called the decay time.



Charging a Capacitor

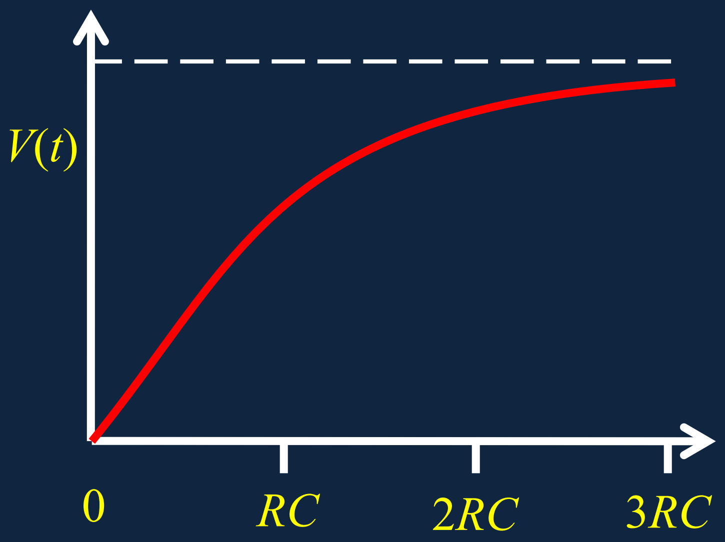
- On closing the switch, charge will flow from the battery into the initially empty capacitor. But how fast?



$$\mathcal{E} = RI + Q/C, \quad I = dQ/dt$$

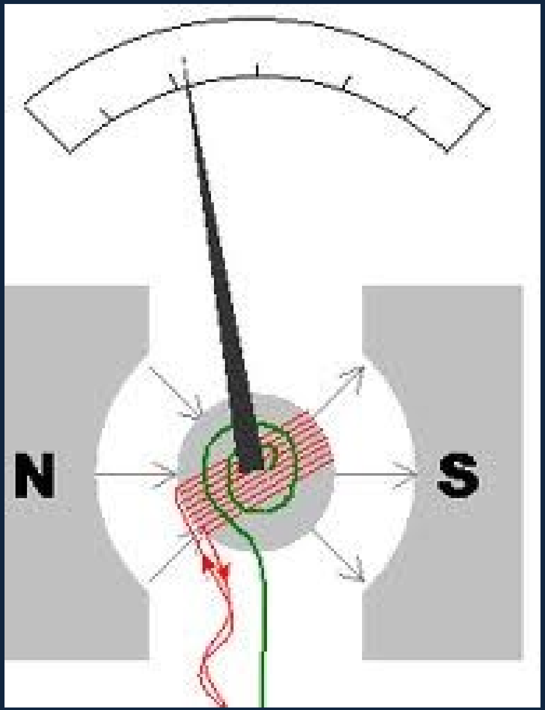
$$dQ/dt = -Q/RC + \mathcal{E}/R$$

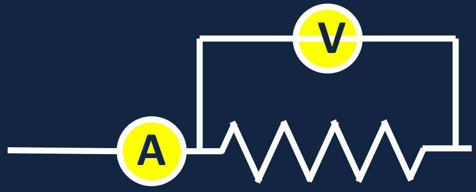
$$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$



Voltmeters and Ammeters

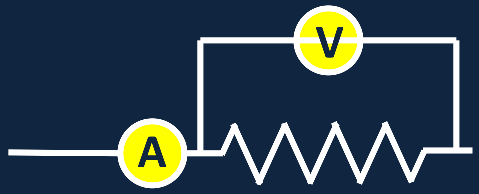
- Historically, voltmeters and ammeters passed current through a small coil between the poles of a magnet, the coil being free to turn, but against a small spring. The coil formed an electromagnet when current flowed.
- An ammeter used a low resistance coil, all the current flowed through it.
- A voltmeter used a high resistance coil, in parallel with the main circuit.





Voltmeters and Ammeters

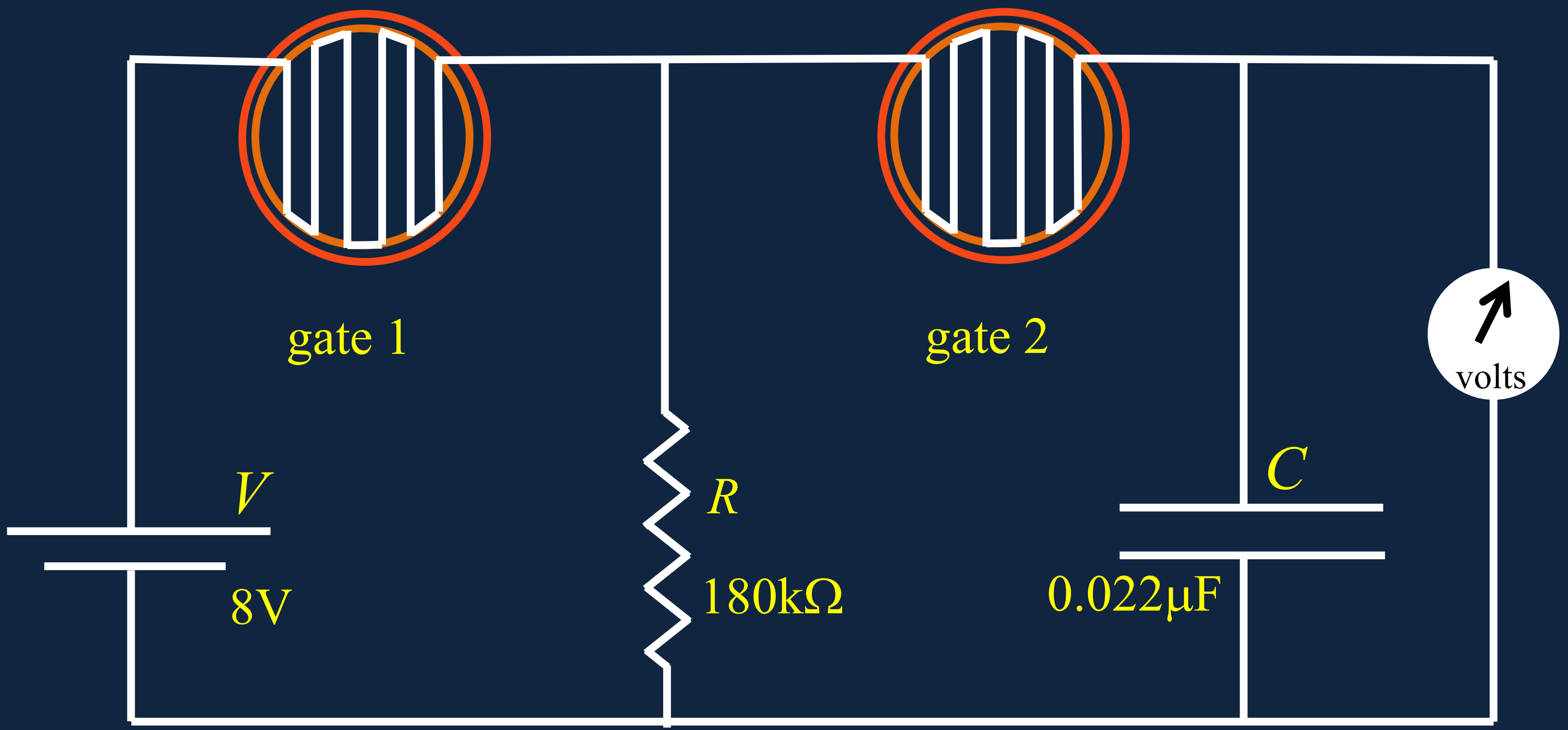
- An ammeter used a low resistance coil, all the current flowed through it: so inserting the ammeter to measure current slightly reduced the current!
- A voltmeter used a high resistance coil, in parallel with the main circuit. Inserting the voltmeter in parallel reduced the resistance, and therefore the voltage measured.



- These meters are in fact obsolete—they've been replaced with DA converters, which affect the quantities being measured far less.

RC Bullet Speed

- As bullet penetrates gate 1, battery is cut out, capacitor C begins to discharge through R: voltage across capacitor $V(t) = V_0 e^{-t/RC}$.
- As bullet goes through gate 2, capacitor is isolated, discharge stops.
- The final voltage gives bullet time from gate 1 to gate 2.



RC Bullet Speed: the Calculation

- As bullet penetrates gate 1, battery is cut out, capacitor C discharges through R: voltage $V(t) = V_0 e^{-t/RC}$.
- As bullet goes through gate 2, capacitor is isolated, discharge stops, voltage gives time from gate 1 to gate 2.
- Taking the initial voltage to be 8V and the final reading to be 4.6V, we have

$$e^{-t/RC} = V / V_0 = 4.6/8 = 0.58.$$

- From the diagram,

$$R = 180\text{k}\Omega,\ C = 0.022\mu\text{F, so } RC = 4.0 \times 10^{-3}\text{ sec}.$$

- So $$-\frac{t}{4 \times 10^{-3}} = \ln 0.58 = -0.55, \quad t = 2.2 \times 10^{-3}\text{ sec}.$$
- This gives a bullet speed of about 450m/sec.