

Magnetism III

Physics 2415 Lecture 16

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Today's Topics

- Electric motors and galvanometers
- Ratio of charge to mass for an electron
- Hall Effect
- Mass spectrometry

Reviewing Essential Facts:

- The force on an element of current in a wire in a magnetic field is:

$$\vec{dF} = I d\vec{\ell} \times \vec{B}$$

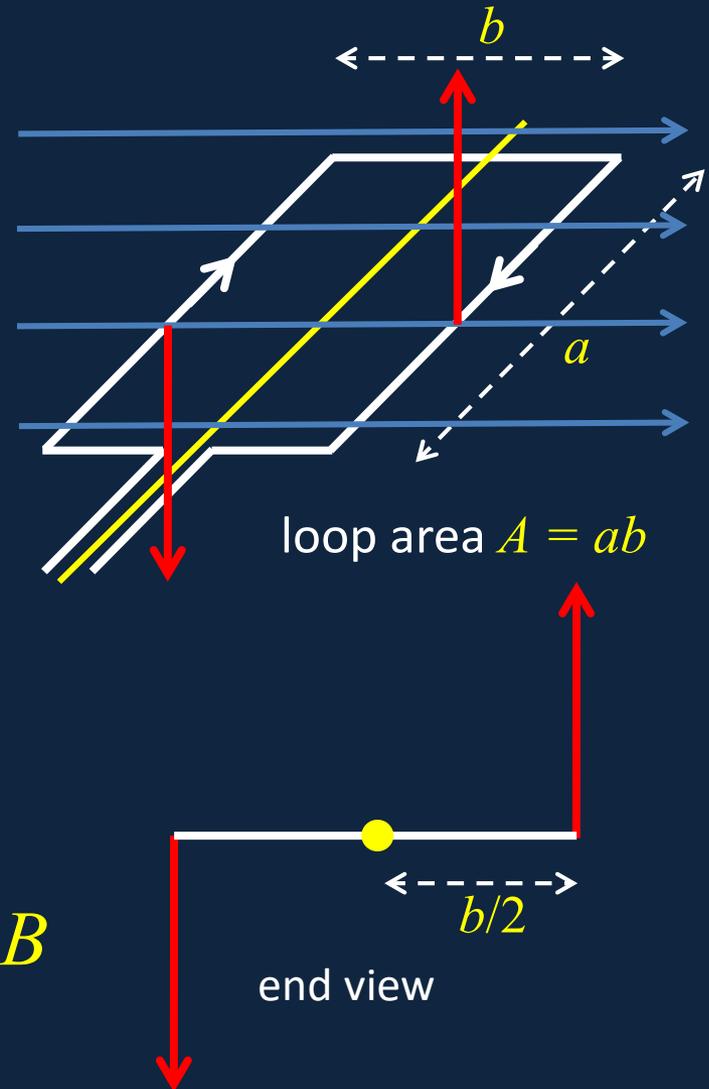
- A charged particle moving through a magnetic field feels a force:

$$\vec{F} = Q\vec{v} \times \vec{B}$$

Torque on a Current Loop

- Take first an $a \times b$ rectangular loop, horizontal, in a uniform magnetic field with field lines parallel to the end sides of the loop.
- The forces on the other sides are vertical as shown, with magnitude $I\ell B = IaB$, and torque about the **axis**:

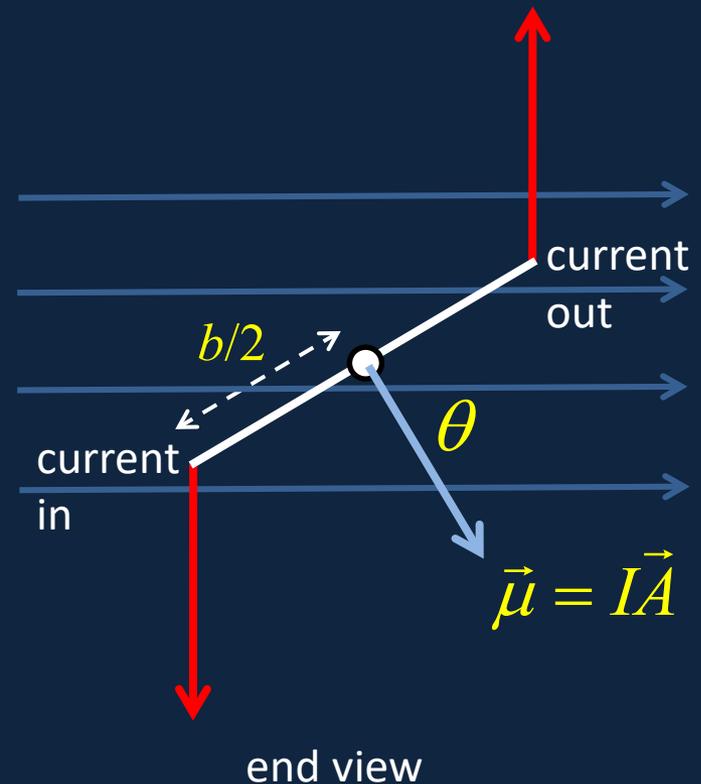
$$\tau = IaBb / 2 + IaBb / 2 = IabB = IAB$$



Current Loop at an Angle

Note: for a coil with N turns, just multiply the single-loop result by N .

- The loop has a magnetic field resembling that of a short bar magnet, we define the direction of the loop area \vec{A} as that of the semi equivalent bar magnet.
- The torque is
$$\tau = IAB \sin \theta = \vec{\mu} \times \vec{B}, \quad \vec{\mu} = I\vec{A}$$
- $\vec{\mu} = I\vec{A}$ is the magnetic dipole moment and this formula is good for round loops too.



Current Loop Potential Energy

- The torque is

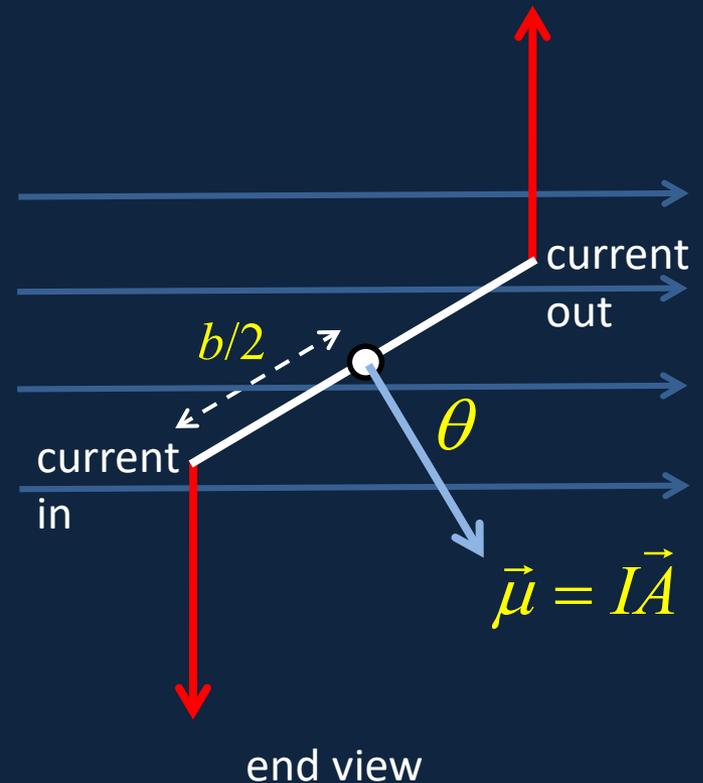
$$\tau = IAB \sin \theta = \vec{\mu} \times \vec{B}, \quad \vec{\mu} = I\vec{A}$$

- is the magnetic dipole moment.

- The work needed to turn the loop is

$$\begin{aligned} U &= \int \tau d\theta = \int IA \sin \theta d\theta \\ &= -\mu B \cos \theta = -\vec{\mu} \cdot \vec{B} \end{aligned}$$

- taking the zero potential energy to be at $\theta = \pi / 2$.



More about the tiny electric motor... and how to make one!

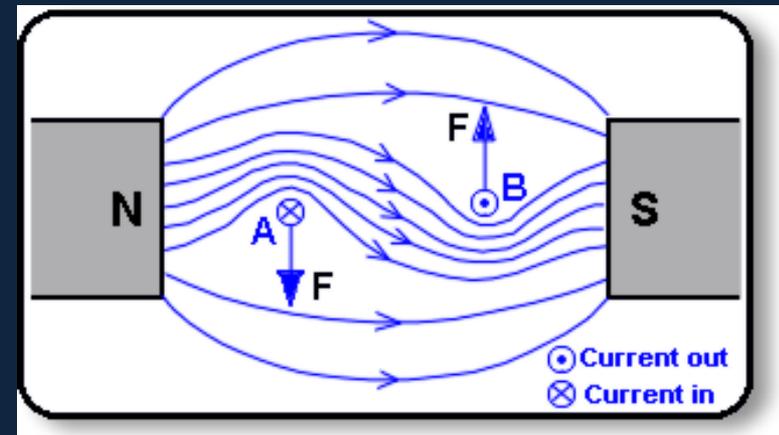
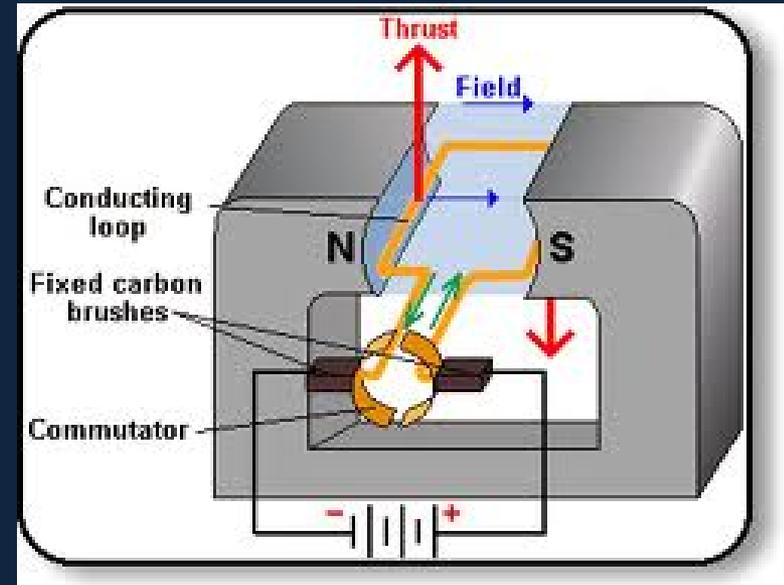
Check these out—impress your friends—prove you're a cool engineer.

- <http://www.youtube.com/watch?v=VhaYLnjkf1E&feature=fvsr>
- <http://www.youtube.com/watch?v=kDxc3QLke1k&NR=1>
- http://www.youtube.com/watch?v=sKzsrJ_Lrb4&feature=related

Basic Electric Motor

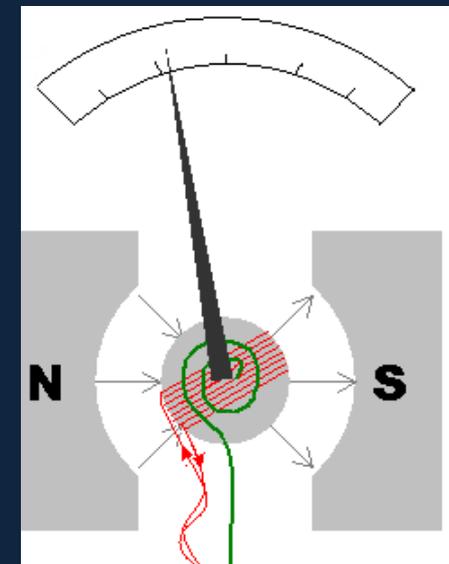
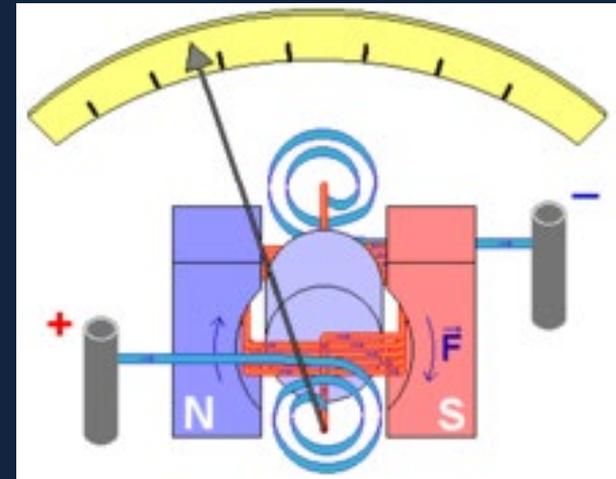
Animation!

- It's just the loop in a magnetic field again, **but with one crucial addition: the commutator**.
- As the loop rotates (think of it as a short bar magnet attracted towards the poles of the big magnet) the commutator **switches current direction**, and therefore switches the loop's poles.
- Faraday pictured the magnetic field lines as elastic, naturally trying to shorten themselves (and repelling each other sideways): helps explain force.



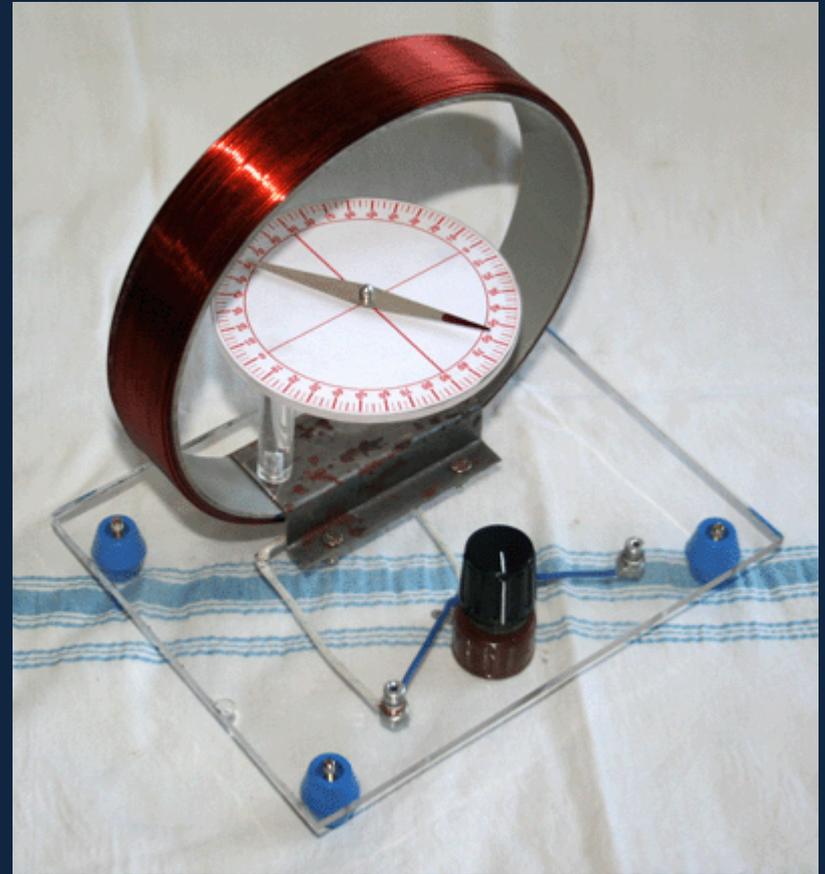
Galvanometers

- Galvanometers measure electric current.
- This one is the familiar loop (but with n turns) in a field between curved poles, with a fixed iron cylinder inside it to give a magnetic field of constant strength, and always perpendicular to the loop's **area vector**.
- The spring exerts a restoring torque proportional to angle.



Tangent Galvanometer

- The pointer is essentially a long compass needle at the center of a fixed loop.
- The loop is aligned with its axis east-west, so the horizontal magnetic field from its current is perpendicular to the horizontal component of the Earth's field, and the ratio of field strengths is the tangent of the needle's angle.



Cathode Rays

- If a high voltage is maintained between an anode (a + charged metal object) and a cathode (-) in a closed glass container, and the air is removed, it is found that “rays” shoot in straight lines outward from the anode, leaving a shadow of objects in the way.
- These “rays” are electrons.

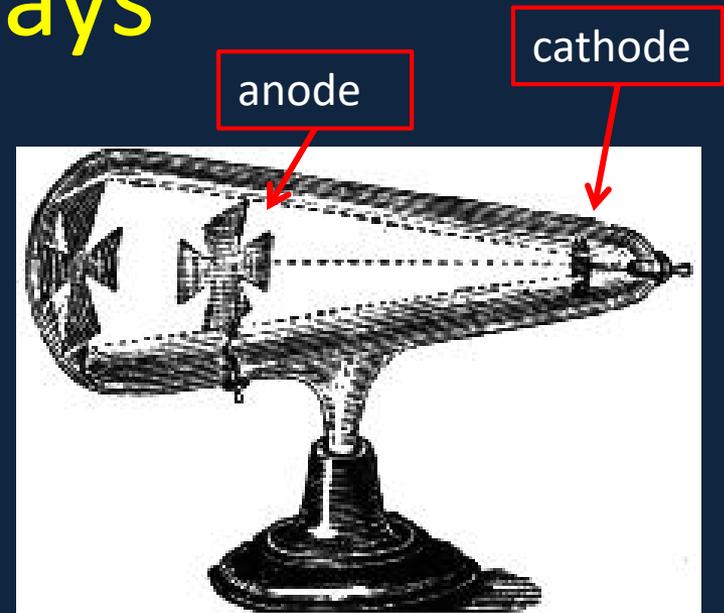
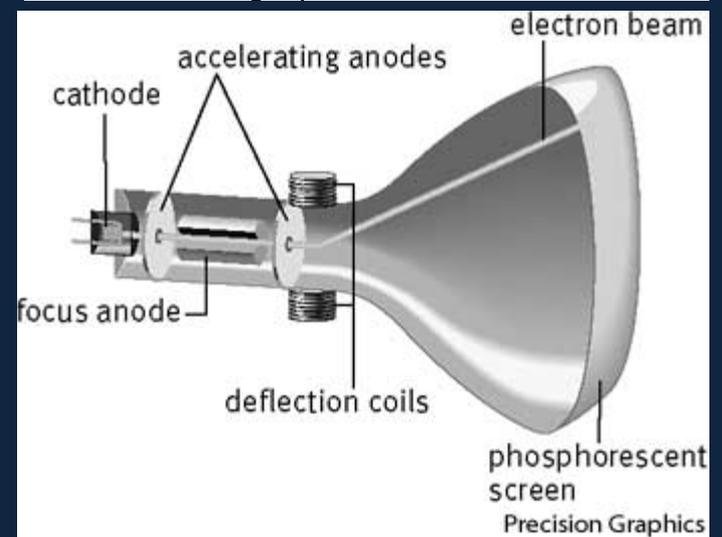
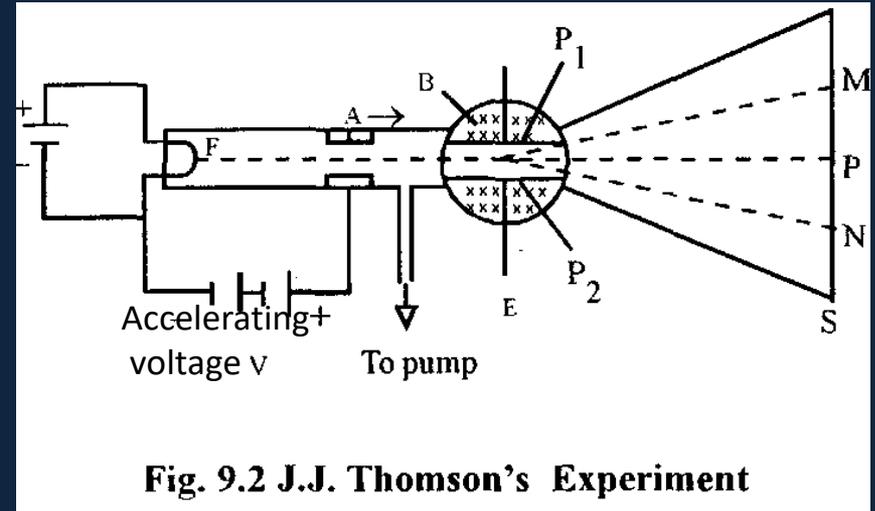


Fig. 515 Linear propagation of cathode rays; shadow formation



Charge/Mass Ratio e/m for Electron

- Heated wire cathode F emits electrons easily: they are accelerated through V.
- Here P_1 and P_2 are parallel plates like a capacitor to produce a **constant vertical electric field E**.
- At the same time, vertical coils produce a **horizontal magnetic field B**, perpendicular to the path of the electrons.

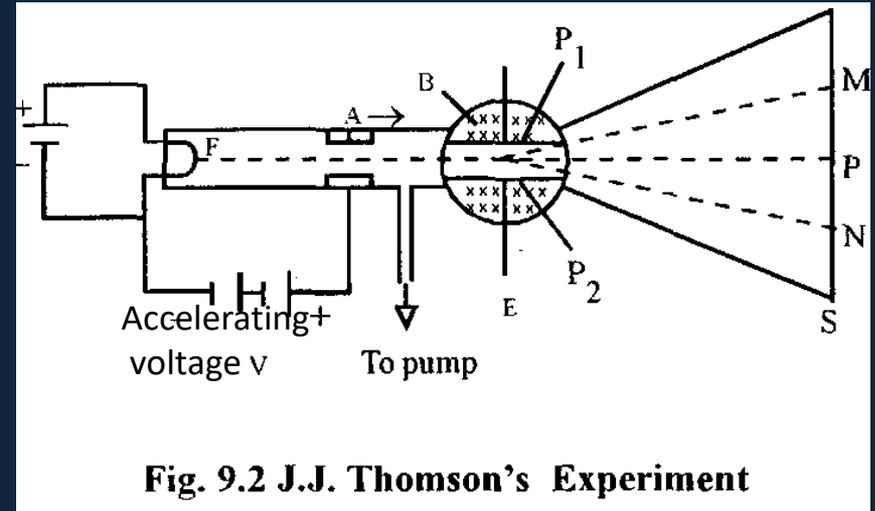


The electrons will not be deflected if $eE = evB$. By suitably adjusting these fields, speed $v = E/B$ can be measured.

Then use **conservation of energy**:
 $eV = \frac{1}{2}mv^2 = \frac{1}{2}m(E/B)^2$, since we know E , B and V , this gives us e/m .

Charge/Mass Ratio e/m for Electron

- Thomson found that e/m for the cathode ray particles (electrons) was about **2,000 times greater** than for the lightest atom!
- Independently it was found that e was similar to the value for atoms, so electrons had to be far smaller than atoms—the **first sign that atoms were made up of smaller things.**



But how do we find e ?

Millikan's Oil Drop Experiment

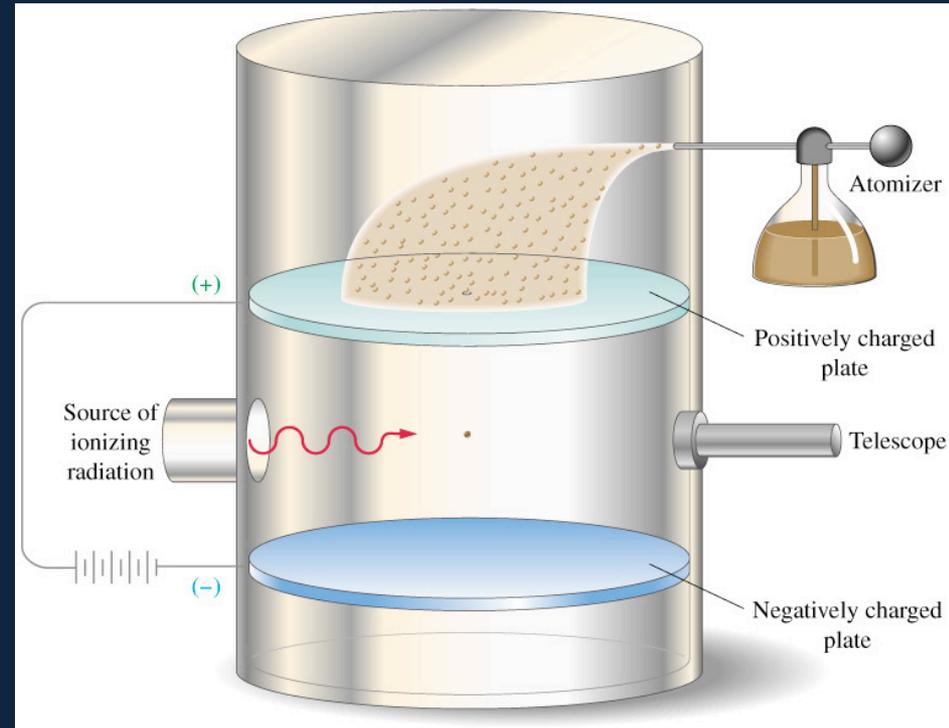
An atomizer generates a cloud of tiny oil drops in the upper chamber: some drift down under gravity into the lower chamber, where there is some ionizing radiation, some drops lose electrons.

Gravity can be balanced by the electric field:

$$Mg = qE$$

M is the mass of the oil drop,

$$M = \frac{4}{3} \pi r^3 \rho.$$

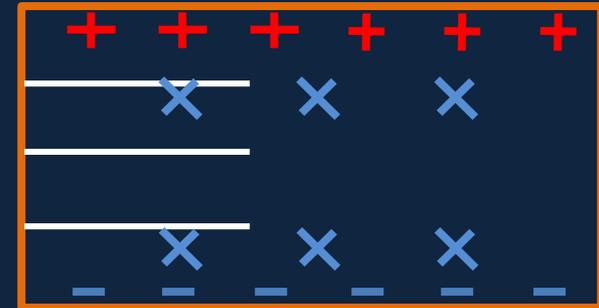
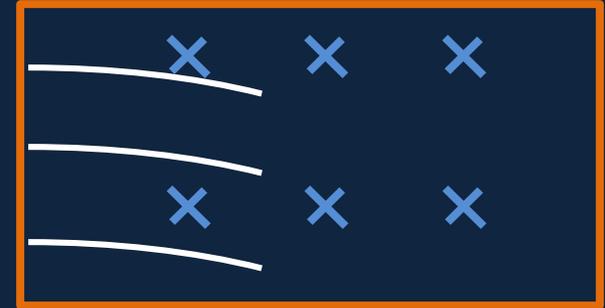


<http://www.youtube.com/watch?v=XMfYHag7Liw>

The experimentally measured charges were all multiples of 1.6×10^{-19} coulombs!

The Hall Effect

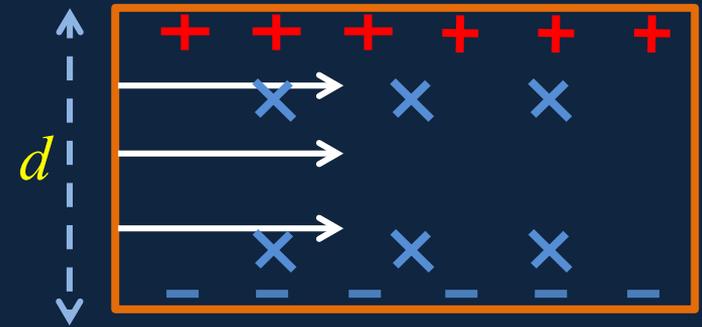
- The force on a current-carrying wire in a magnetic field is really **a force on the current**—the moving charged particles. *How does the force get transmitted to the wire?*
- Initially, the charges go in curved paths, but then charge piles up along the edges of the wire until an electric field balances the magnetic force, and the charges go straight. But by Newton's third law, the moving charges are now pushing the wire sideways.



The Hall Effect

- Imagine a rectangular wire, thickness d , magnetic field B into the screen, negative charges streaming in from the left. For charge (drift) velocity v_d , the charge layers along the top and bottom must generate an electric **Hall field** $E_H = v_d B$ for the current to go straight.
- The total voltage drop from top to bottom is called the **Hall emf**,

$$\mathcal{E}_H = E_H d = v_d B d$$



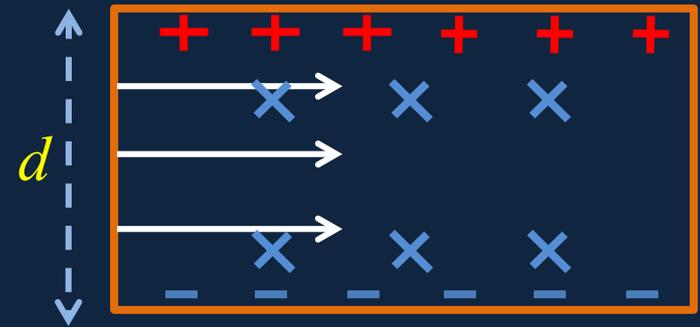
Don't confuse the Hall field E_H and the Hall emf \mathcal{E}_H !

The Hall Effect : Question!

- Hall did his experiment (at Johns Hopkins) in 1879, well before the discovery of the electron, and no-one knew if an electric current was negative charges moving one way or positive charged particles moving the other.
- Can this experiment distinguish between these two theories?

A. Yes

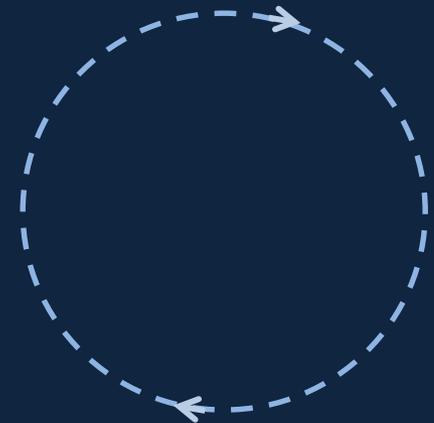
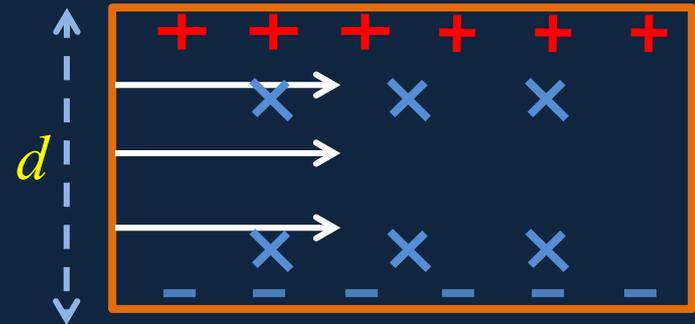
B. No



The Hall Effect: Answer

- No-one knew if an electric current was negative charges moving one way or positive charged particles moving the other.
- Can this experiment distinguish between these two theories?

A. Yes: if negative charges moving to right curve down, one moving to the left would move up (both paths parts of a clockwise circle), so **positive** charges moving left go down, **+**'s will go to the **bottom**.



Path of negative charge
in B into screen

Finding the Drift Velocity



- Suppose a **one amp** current is flowing down a wire of cross-section area A . Take a **one meter length** of the wire: it has volume A cubic meters. If the conductor has density ρ , the mass of this 1 meter is ρA . If the conductor has atoms of mass m , each contributing one conduction electron, the total conduction electron charge in 1 meter is $\rho A e / m = n$ Coulombs, say. The drift speed for 1 amp is then the *inverse* of this number: $v_d = 1/n$.

Mass Spectrometers

- A beam of light can be broken down into its component colors (wavelengths) by a spectrometer, such as a prism or diffraction grating. The collection of colors is the “spectrum”.
- Similarly, a beam of charged atoms or molecules can be separated into different **masses** by various devices using electric and/or magnetic fields. Such a device is called a **mass spectrometer**.

Time of Flight Mass Spectrometer

- The TOF idea is simple: the ionized molecule is accelerated through a known potential difference V , then its speed v is measured accurately. This gives e/m . It works for atoms but also for proteins, etc.

$$eV = \frac{1}{2}mv^2$$



Using Magnets to Separate Ions by Mass

same idea, just up to date...

