home

Physics 2415 Lecture 20: Magnetic Induction II

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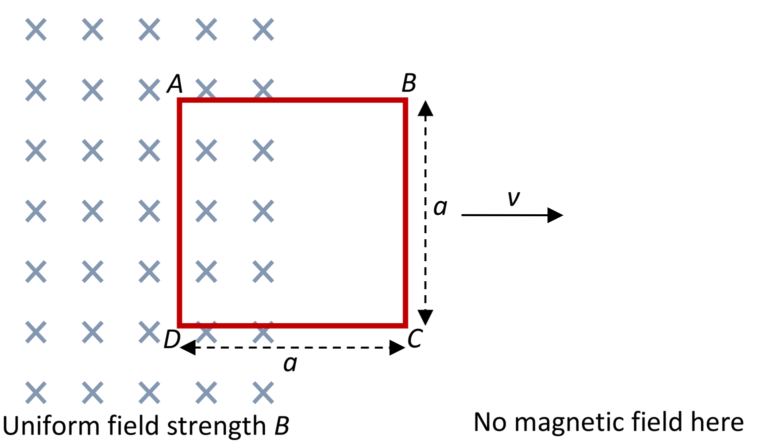
Pulling a Square Loop out of a Magnetic Field

Let's assume for purposes of illustration that we have a magnetic field pointing inwards, and it ends suddenly at the plane $x=0$ (which has to be an approximation, since Ampere's Law wouldn't be satisfied integrating $\vec{B}\cdot\vec{d\ell}$ around a loop perpendicular to this one—but it could be a good approximation. For example, we could be pulling the loop out of the strong field between the poles of a permanent magnet).



Pulling a square conducting loop out of a uniform magnetic field *B* at speed *v*

From Faraday's law, we know the emf $\mathcal{E}$ induced is

$\mathcal{E}=-d\Phi_B/dt$, so in this case $\mathcal{E}=avB$, and the current direction from Lenz' law is clockwise, to generate inward magnetic flux to replace some of that lost.

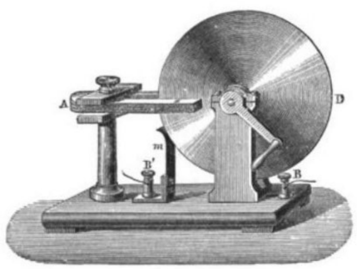
There's another way to understand this: consider the electrons in the leg *AD* of the loop. As a result of the loop being pulled sideways at speed *v*, they will feel a Lorentz force $\vec{F}=q\vec{v}\times\vec{B}$, driving them down the wire (since they're negatively charged). And since the leg *BC* is out of the field, there is no balancing force to prevent a current from flowing. Relative to the electrons in the wire, the *moving* magnetic field has generated an electric field along the wire of strength *vB*, corresponding to a potential difference, or emf, of *avB*.

*(*Footnote for anyone interested*: In fact I think this argument works in the general case. Visualizing the magnetic field a la Faraday, in terms of lines of force, suppose any loop of wire has the magnetic flux linking through it changing, because of the loop contorting, moving, or the field changing, or whatever. It seems clear from the Faraday picture that flux can only become unlinked by moving *across the wire* (which itself may be just a mathematical curve). If the relative motion of an infinitesimal part of the loop and the local magnetic field $\vec{B}$ is $\vec{v}$, there is an increment of potential from the Lorentz force $d\mathcal{E}=\vec{v}\times\vec{B}\cdot d\vec{\ell}$, and writing $\vec{v}=d\vec{x}/dt$ this is

$$d\mathcal{E}=\frac{d}{dt}\left(d\vec{x}\times\vec{B}\cdot d\vec{\ell}\right)=\frac{d}{dt}\left(d\vec{\ell}\times d\vec{x}\cdot\vec{B}\right)=\frac{d}{dt}\left(d\vec{A}\cdot\vec{B}\right)$$

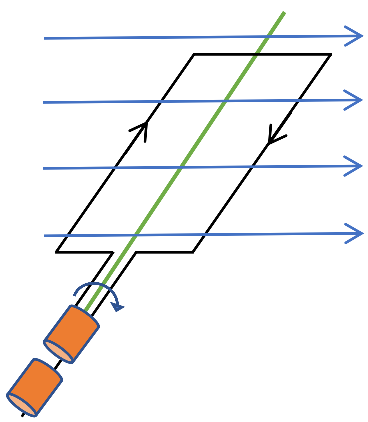
where $d\vec{A}$ is the element of area swept out by the wire's movement relative to the magnetic field. But $d\vec{A}\cdot\vec{B} = d\Phi$, the change in enclosed flux caused by this movement. One can write this $d\mathcal{E} = \vec{E}\cdot d\vec{\ell}$ where $\vec{E}$ is the electric field in the wire's frame of reference, and, adjusting signs suitable using Lenz' law, this gives $\int \vec{E}\cdot d\vec{\ell} = -d\Phi_B / dt$. What about the uniform magnetic field between close flat poles of a large electromagnet, when the current is increasing? I would visualize that as magnetic field flowing radially inwards, find the appropriate velocity, etc.)

Electric Generators



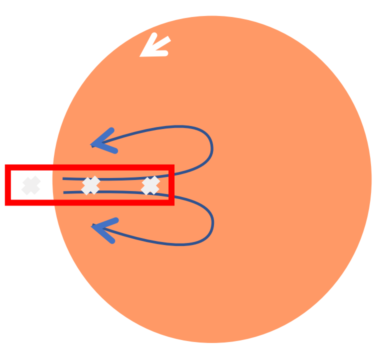
Here is a copy of the first electric generator, constructed by Michael Faraday in 1831. A is the magnet; B, B' the terminals. On rotating the copper disc, an emf is generated in the region between the poles of the horseshoe-like magnet A. Taking that field to be into the screen, and the disc spinning anticlockwise, an electron in the disc will feel a radial electric field. The circuit is completed by having an outside wire from the axle to the outside of the disc.

Conceptually, the simplest generator is a single loop of area A rotated at constant angular speed ω in a constant uniform magnetic field $\vec{B}$, the axis of the loop being perpendicular to $\vec{B}$. The magnetic flux through the loop, as previously discussed, is $\Phi_B(t) = AB\cos\omega t$ so the induced emf is



$\mathcal{E}(t) = AB\omega\sin\omega t$. This is an AC (alternating current) generator. If the two ends of the loop are connected via slip rings to an external resistive circuit, current will flow. This current will of course give the loop a dipole like magnetic field, which from Lenz' law will be such as oppose the imposed rotation. This means that once the circuit is complete, whatever is maintaining the rotation will have to work harder, obvious from energy conservation, since the circuit is now generating heat in its wires, and possibly useful work as well. It's worth checking with a small generator, such as for lights on a bicycle, how much harder it is to turn when connected to a circuit.

Eddy Currents



In fact, any time magnetic field strength is changing, circling electric fields are generated, and if these fields are in a conducting medium, currents will arise creating magnetic fields partially compensating for the changes taking place in the original magnetic field. These currents are called eddy currents—they are reminiscent of the circling eddies in the wake of a boat, since they are generated by the nonconservative circling electric fields. Notice the opposite directions of the eddy currents—on entering the magnetic field, the currents oppose the field increase, on leaving, they attempt to maintain the field strength. Induction cookers use eddy currents.