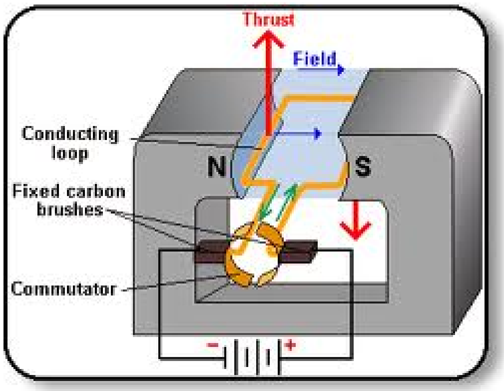
home

Physics 2415 Lecture 21: Magnetic Induction III

Michael Fowler UVa

Electric Motors



One common electric motor design is just the "loop in a magnetic field" generator we already discussed, but now run backwards—by which we mean power is supplied from a battery, say, to the loop which then becomes a dipole-like electromagnet. The loop is between the poles of a permanent magnet so it turns, but as it reaches the limit a commutator reverses its current supply, so attraction suddenly becomes repulsion and since it's rotating it continues around, whereupon the sequence repeats.

Back emf

As the loop rotates in the magnetic field*, that rotation will induce an emf in the loop opposing the motion*—in other words, opposing the driving emf!
This is called *back emf*, and is proportional to speed.

When a motor is first connected, it is not turning and Ohm's law gives $V_0 = IR,$ where V_0 is the voltage of the supply, and R the resistance of the armature (meaning the loop or coil). Heat production inside the motor is $I^2R.$

When the motor is running under load, there is a back emf $V_{\text{back}},$ and now $V_0 - V_{\text{back}} = IR.$

Heat production in the motor is now $I^2R:$ which can be *much less* than it was initially.

If a blender is mechanically overloaded so the motor turns slowly, back emf is small, the current is higher than designed for, high heat production for some time may cause burnout.

Back emf problem:

A motor has an armature resistance of 4Ω.

It draws 10A from a 120-V line when running at its design speed of 1000 rpm.

If a load slows it to 250 rpm, what is the current in the armature?

Counter Torque

A generator is essentially a loop rotating in a magnetic field.

If the generator is connected to an outside circuit, the induced emf will cause a current to flow: that's the point of the generator!

But the current carrying wire moving through the field will feel Lenz-type forces opposing its motion: called the “counter torque”.

So to produce a current through the external circuit work must be done. Obviously.

An example from the book: #34.

A conducting rod, mass *m*, resistance *R*, length ℓ, rests on two frictionless and resistanceless parallel rods, in a perpendicular magnetic field *B*. At *t* = 0, a source of emf is supplied to the rails. How does the rod move, if case (a) the source maintains constant current *I*, and case (b) the source supplies constant emf $\mathcal{E}$?

Case (a): in a magnetic field B, the force on a length of wire carrying a current is $\vec{F} = I\vec{\ell} \times \vec{B}$, so if *B* is inwards, and the current is flowing downwards (top rail positive) the rod will feel a constant force $I\ell B$ to the right, and hence accelerate at a uniform rate.

Case (b): the rod will still feel accelerating force $I\ell B$, but now the current will vary because the motion of the rod through the field generates an emf in the rod, of magnitude $\mathcal{E} = -d\Phi_B / dt = -v\ell B$. You can check that this emf is opposing the driving emf: the rod is moving to the right, so the force on a charge *q* in the rod is $q\vec{v} \times \vec{B}$, upward for a positive charge, opposing the external voltage. But this is also just Lenz’ law: the induced emf is such as to oppose the motion.

Now, the current I is given by $I = \left(\mathcal{E} - v\ell B\right) / R$, and the force accelerating the rod is $I\ell B$, so

$$m\frac{dv}{dt} = I\ell B = \frac{\left(\mathcal{E} - v\ell B\right)\ell B}{R}$$

From which

$$\frac{dv}{\mathcal{E} - v\ell B} = \frac{\ell B}{mR}dt$$

You may recognize the form of this equation: it’s the same as the one we found for charging a capacitor. The left-hand side integrates to a log, the right hand side is trivial, a constant times *t*. We’ll leave that as an exercise: the result is just like charging a capacitor, initially the rod accelerates at a constant rate, but there’s a limiting speed, as *v* approaches $\mathcal{E} / \ell B$ the left-hand side blows up, so to match it longer and longer times elapse. The speed approaches this value but never quite reaches it.

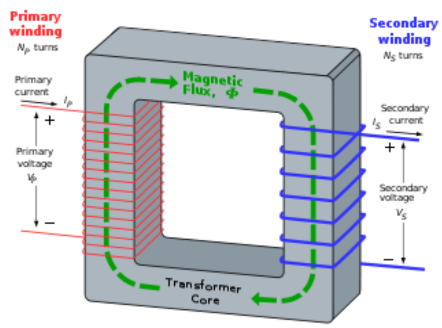
What’s happening to the current in the rod at this stage? It’s getting smaller and smaller, remember $I = \left(\mathcal{E} - v\ell B\right) / R$.

This is an example of back emf: if an external voltage is used to supply a current to a moveable wire, which is in a magnetic field and moves because of the Lorentz force, then the magnetic field induces an emf in the conductor opposing the supplied voltage. Again, this is just Lenz’ law.

Back emf plays an important role in electric motors, and can be a big fraction of the applied emf. Of course, there is no back emf in a jammed motor, and this is why a jammed motor will likely burn out—

the coils are only designed to take the large current from the unopposed external emf for a short time, the design assumes there will be back emf when the motor is in use, limiting the current.

Transformers



The big reason power is transmitted as ac rather than dc is that it's very easy to change the voltage using transformers for ac, and much less power is lost in transmission lines that run at high voltage. A transformer has two solenoid-type coils: the input power goes into the primary, producing a constantly changing magnetic field, this changing flux induces oscillating emf in the secondary coil. To maximize the effect, soft iron or some similar alloy is used to guide all the flux from the primary through the secondary. This iron is laminated—it's in thin sheets separated by very thin layers of insulation, to minimize eddy current buildup.

Neglecting energy loss from eddy currents and Ohmic loss in the coils themselves, the back emf in the primary must be balancing the input voltage, and this back emf equals $N_P d\Phi_B / dt$, where Φ_B is the *total* magnetic flux through the coil, this flux coming from both the primary coil *and* the secondary coil if that is part of a circuit. What is the output voltage? The same rate of change of flux takes place in every turn of the secondary (output) coil also, so

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}.$$

The ratio of voltages is just the ratio of the numbers of turns!

The Betatron

This is a very clever device for accelerating electrons to very high speeds (close to the speed of light). It uses a magnetic field to make the electrons go in circles of radius r, as earlier (for the cyclotron) we have

$$mv^2 / r = evB(r), \quad mv = reB(r)$$

where we have a perpendicular magnetic field with strength a function of r, arranged by appropriately shaped poles of an electromagnet.

Now, we increase the magnetic field: this generates a momentary circling electric field,

$$\oint \vec{E} \cdot d\vec{\ell} = -d\Phi_B / dt$$

and for a circular path $2\pi rE = \pi r^2 d\overline{B} / dt$, or $E = (r/2) d\overline{B} / dt$, where $\overline{B}$ is the average magnetic field inside the circle.

In the betatron, the magnetic field is increases in such a way that the electrons continue to circle at the same radius, but are speeded up by the electric field generated by the increasing magnetic field.

How is that possible?

$$\frac{d(mv)}{dt} = E$$

and if r is constant,

$$\frac{d(mv)}{dt} = re\frac{dB(r)}{dt} = eE = \frac{er}{2}\frac{d\overline{B}}{dt}$$

so provided $B(r) = \overline{B}/2$, the electrons will continue to orbit at the same radius! The pole pieces can be designed so this is true at the design radius.

The betatron, unlike the simple cyclotron, can accelerate electrons to relativistic speeds, where they have greatly increased mass.