More Circular Motion

Physics 1425 Lecture 10

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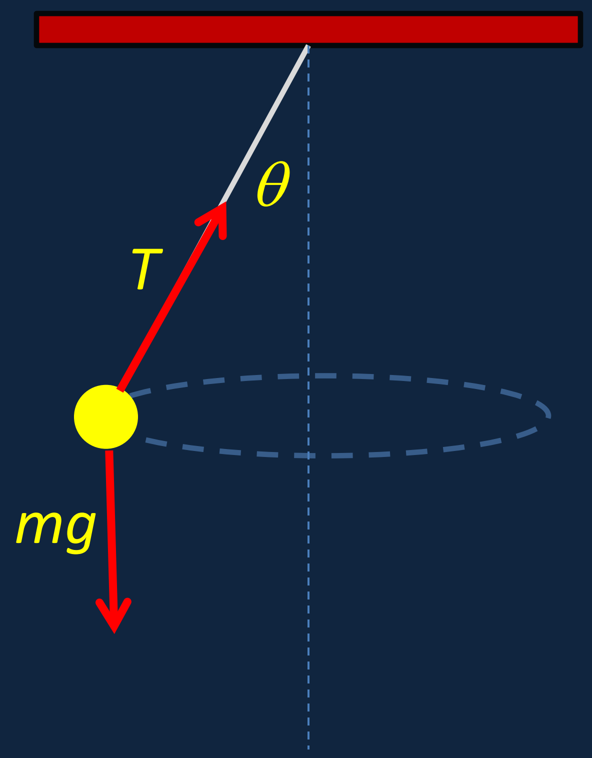
The Conical Pendulum

- A mass moving in a horizontal circle, suspended by a string or rod from a fixed point above.
- If the tension in the string or rod is *T*, and the string is θ degrees from the vertical,

$$T\sin\theta = mv^2/r,$$

$$T\cos\theta = mg,$$

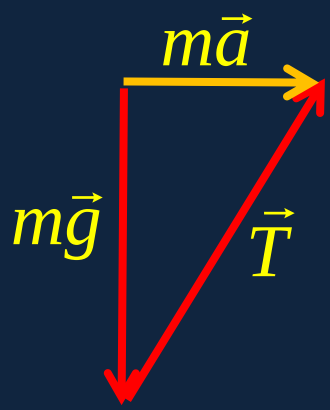
$$\tan\theta = v^2/rg.$$

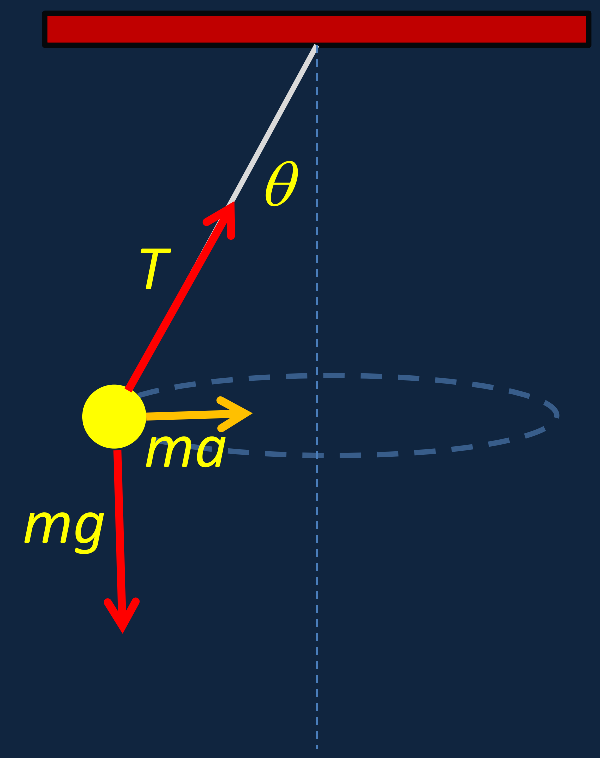


$\vec{F} = m\vec{a}$ for the Conical Pendulum

- Notice how vector addition gives

$$\vec{F} = m\vec{g} + \vec{T} = m\vec{a}$$





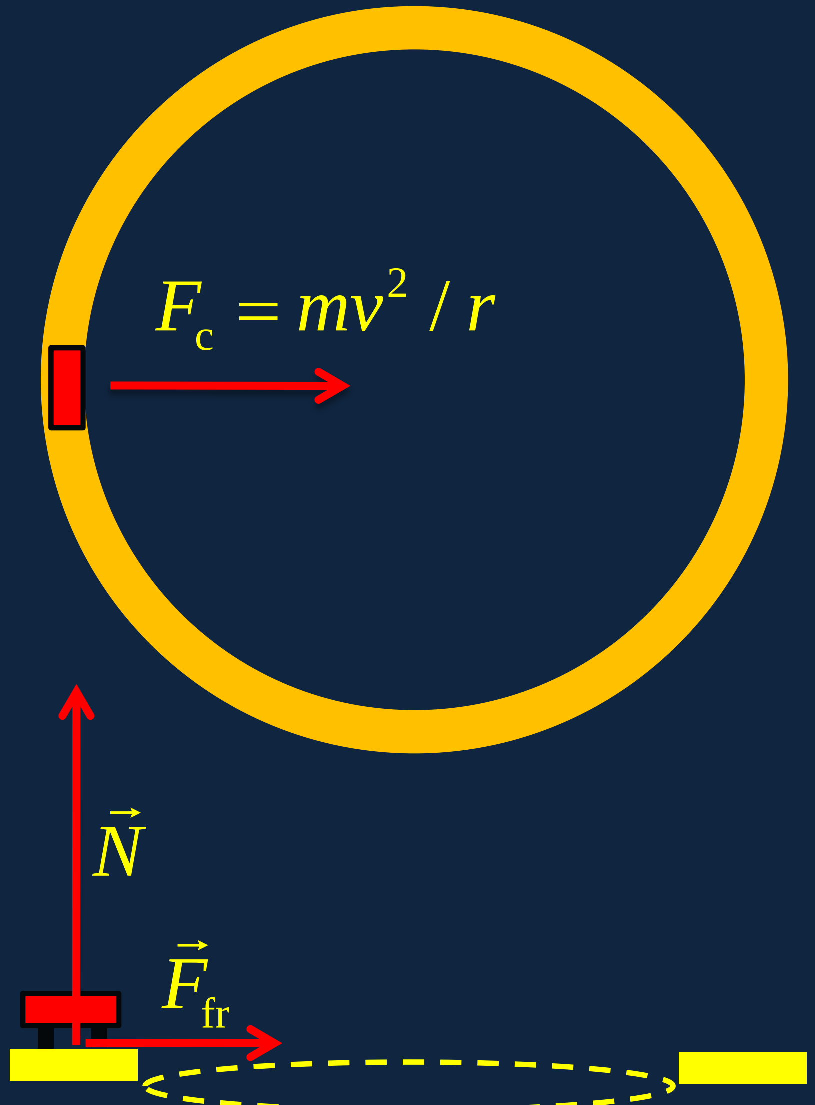
Conical Pendulum as Control

- An early steam engine: as the conical pendulum rotates faster, driven by the engine, the masses rise and the levers cut back the steam supply.
- It can be preset to keep the engine within a given speed range.

Car on Flat Circular Road

- For steady speed *v* on a road of radius *r*, there must be a centripetal force mv^2/r.
- This is provided by friction between the tires and the road: at maximum nonskid speed

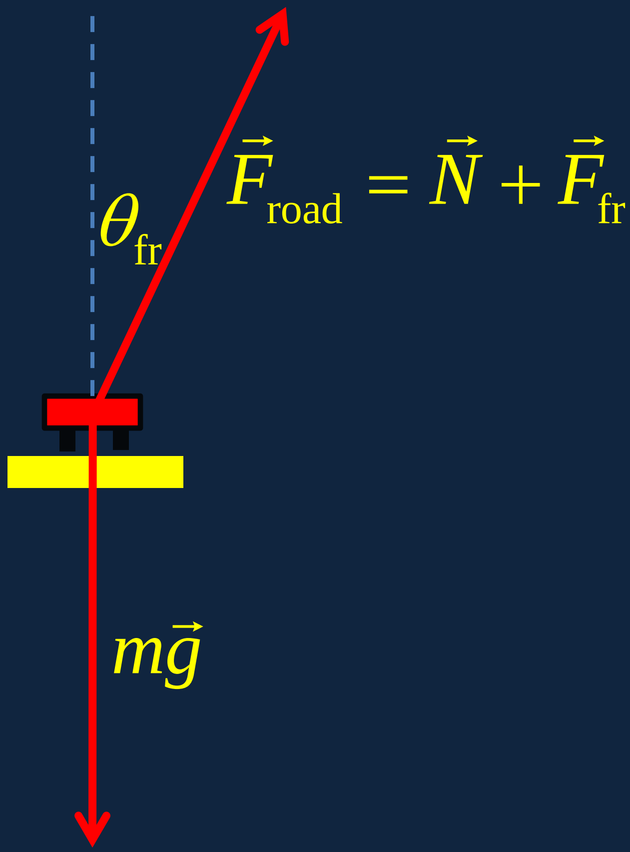
$$F_{\text{fr}} = \mu_s N = \mu_s mg = mv^2 / r$$



Total Road Force on Car

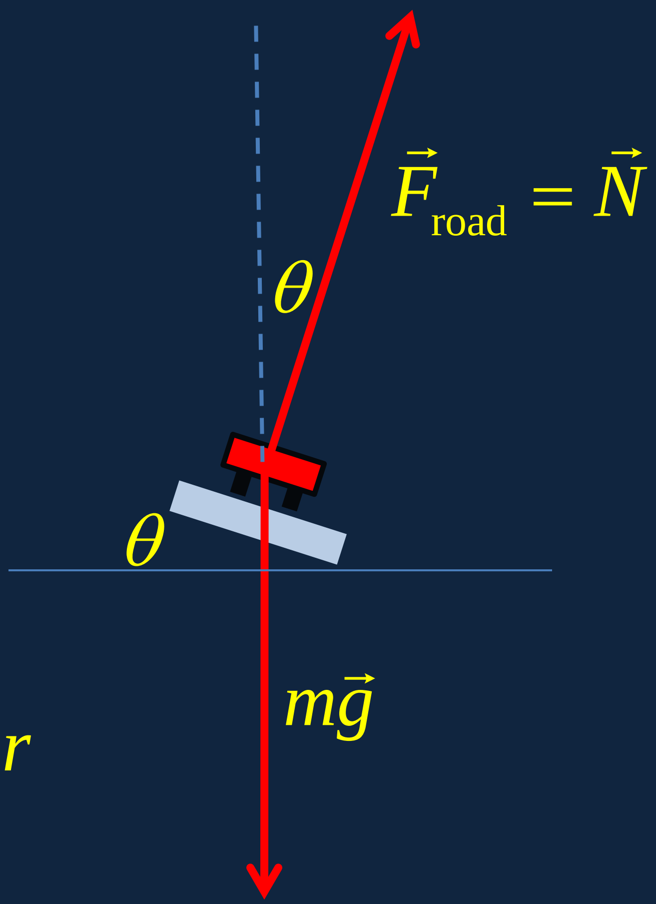
- The actual force $\vec{F}_{\text{road}}$ on the car from the road is the vector sum of the normal force and the frictional force.
- Notice the forces on the car have the same configuration as the conical pendulum!
- At maximum nonskid speed, $\vec{F}_{\text{road}}$ is at an angle θ_{fr},

$$\tan\theta_{\text{fr}} = F_{\text{fr}} / N = \mu_{\text{s}}.$$



Banked Road: Sheet of Ice

- The **normal force** is always **perpendicular** to the road surface.
- Banking a curved road turns $\vec{N}$ inward to provide a centripetal force even at zero friction—but only for the *right speed*!



$$N\cos\theta = mg, \;\; N\sin\theta = mv^2 / r$$

- So $\boxed{v^2 = rg\tan\theta}$ (the same as the conical pendulum)

Maximum Speed on Banked Road

- At maximum speed, friction adds $\vec{F}_{\mathrm{fr}}$ to $\vec{N}$ to give a total road force

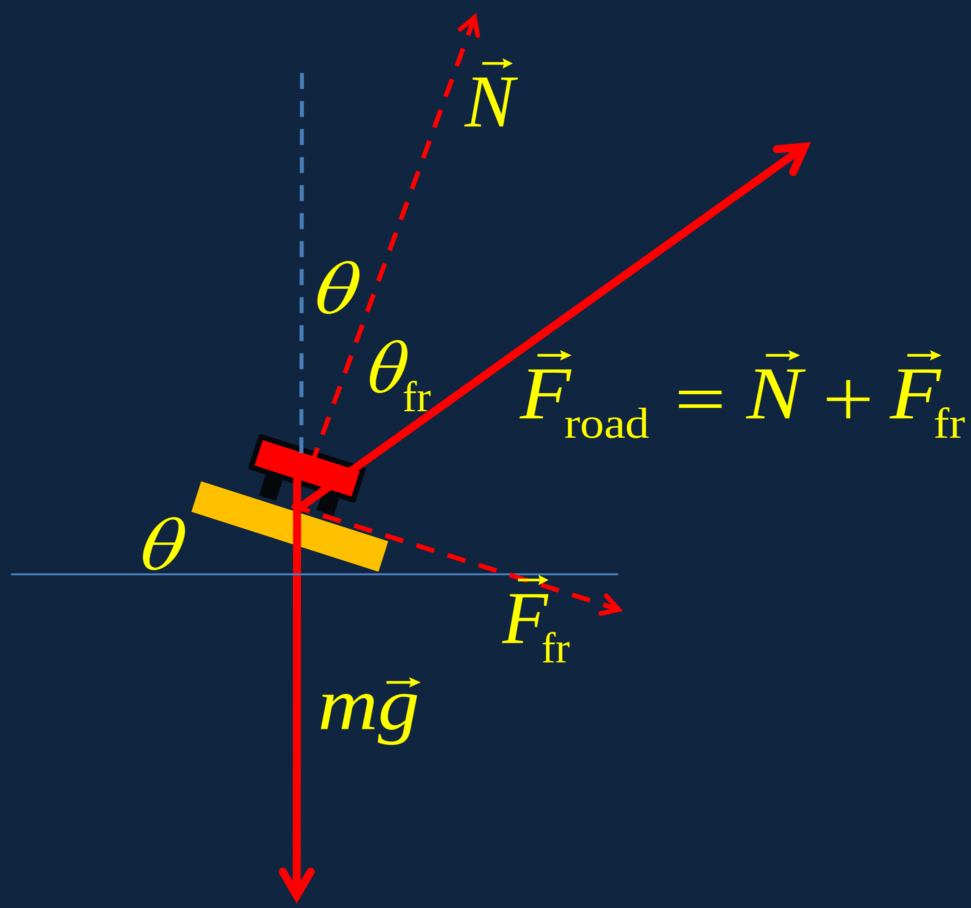
$$\vec{F}_{\mathrm{road}} = \vec{N} + \vec{F}_{\mathrm{fr}}$$

at an angle θ_{fr} to $\vec{N}$, where

$$\tan\theta_{\mathrm{fr}} = F_{\mathrm{fr}} / N = \mu_{\mathrm{s}}.$$

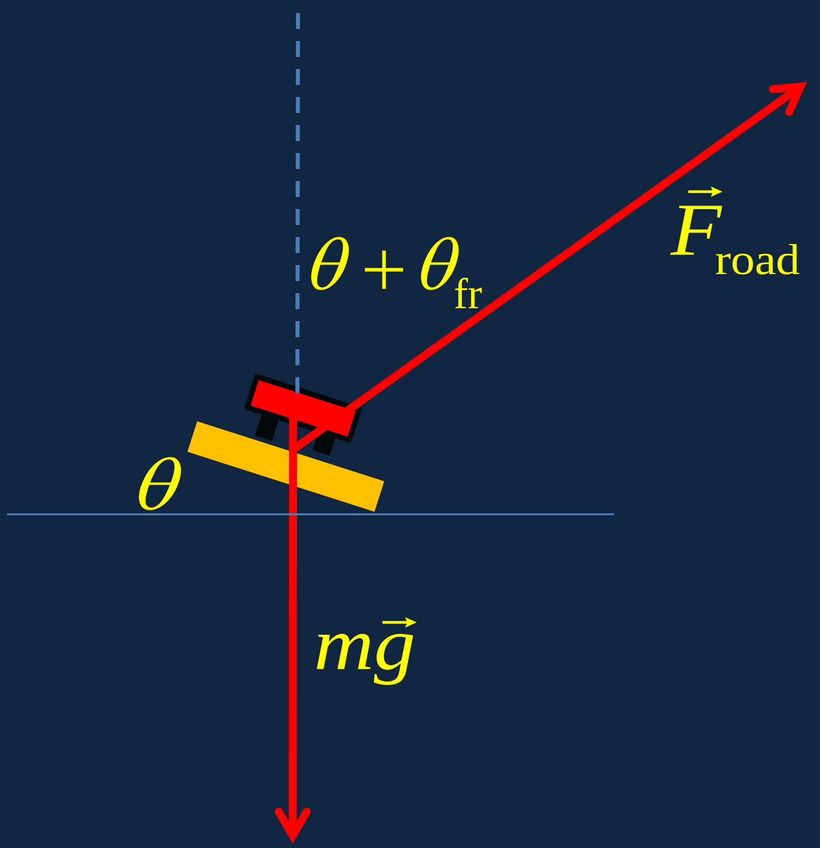
- The only forces acting on the car are $\vec{F}_{\mathrm{road}}$ and $m\vec{g}$, so the conical pendulum equation is correct again:

$$v_{\max}^2 = rg\tan\left(\theta + \theta_{\mathrm{fr}}\right)$$



Maximum Speed on Banked Road

- Here are the two forces acting on the car, $\vec{F}_{\text{road}}$ and $m\vec{g}$.
- Racing tires can have coefficient of friction μ_s close to 1, so from
  $\tan\theta_{\text{fr}} = F_{\text{fr}} / N = \mu_s$,
  θ_{fr} can be 45°.
- Now $v_{\max}^2 = rg\tan(\theta + \theta_{\text{fr}})$, so for *banking angle* 45°, *and* μ_s = 1, $v_{\max}$ is infinite!



(Of course, as *v* becomes *very* large, so does the centripetal force **and therefore the normal force**—something will give!)

Clicker Question

What is the direction of the acceleration of a pendulum at the furthest point of its swing?

A. Downwards.
B. In the direction it’s about to move.
C. No acceleration at this point.

Clicker Question

What is the direction of acceleration of a pendulum at the midpoint of its swing?

A. Downwards
B. Upwards
C. Horizontal
D. No acceleration at this point.

Clicker Question

What is the direction of acceleration of a pendulum halfway down from the furthest point towards the midpoint of its swing?

A. Downwards
B. Upwards
C. Along the path
D. At some angle to the path, pointing above the path.
E. At some angle to the path, pointing below the path.

Nonuniform Circular Motion

- The swinging pendulum is an example of nonuniform circular motion, as is a car picking up speed on a curve.
- Remember acceleration is a vector: it has a component in the direction of motion (called the tangential component) equal to the rate of change of velocity in that direction—the car's acceleration along the road, dv/dt.
- It also has the usual v^2/r centripetal component towards the center of the curve.

Drag Forces

- There are two kinds of drag forces:
- Viscous drag, as in pushing something through molasses. This drag force is linear in *v*. It's relevant for tiny particles in air and water, and small bubbles in molasses, etc.
- Inertial drag: the effort involved in shoving air or water out of the way as you move through it. This is proportional to v^2, and this is the usual drag for cars, boats, etc.
- Terminal velocity: for a falling object, the speed at which the drag force equals *mg*, so no *net* force acts, the object falls at constant speed.