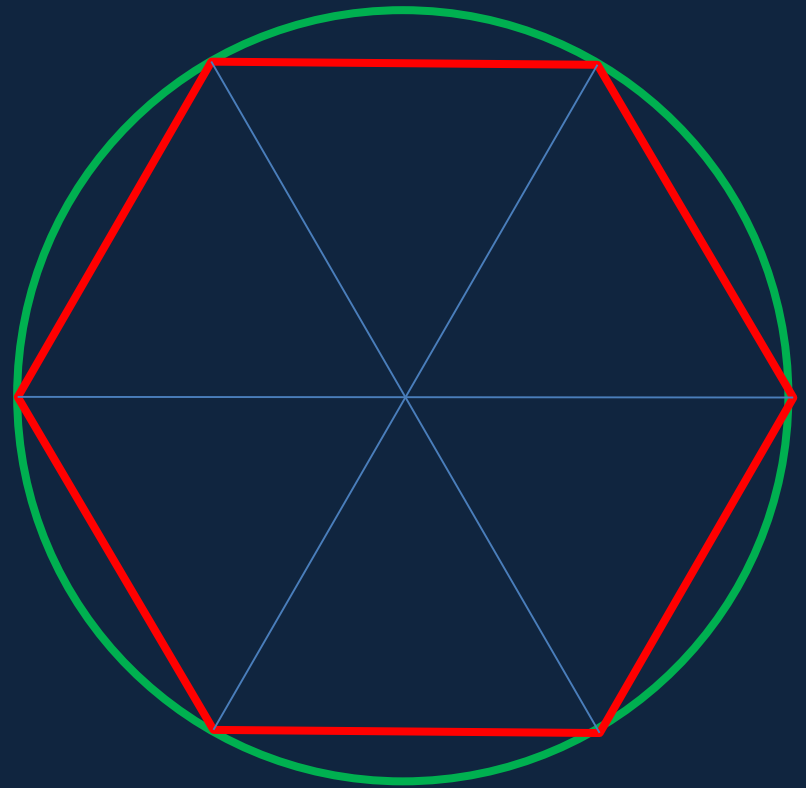


Circular Motion

Physics 1425 Lecture 18

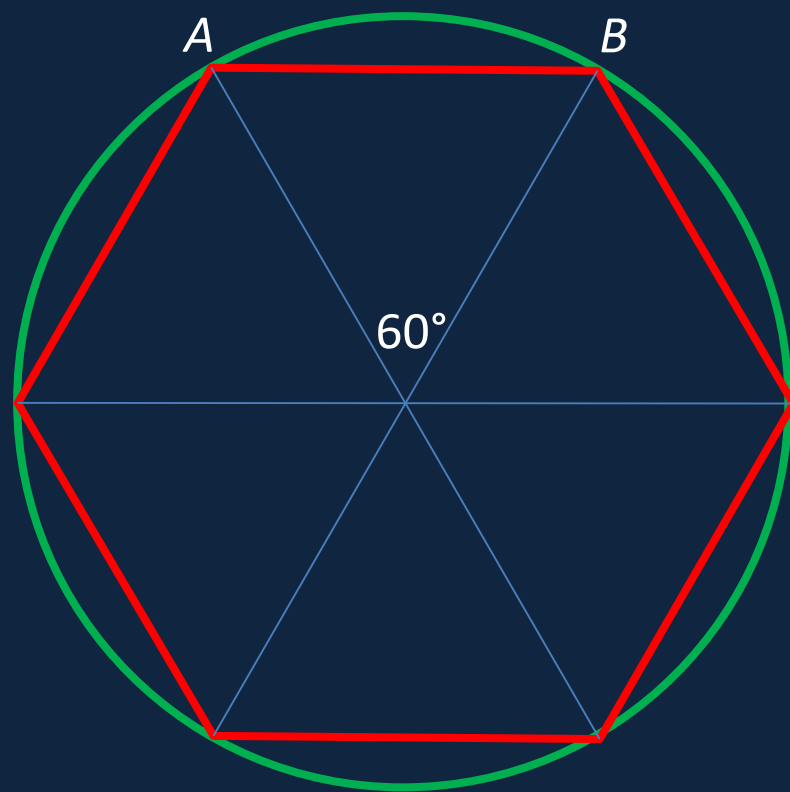
How Far is it Around a Circle?

- A **regular hexagon** (6 sides) can be made by putting together 6 equilateral triangles (all sides equal).
- The radius of the **circle** = 1.
- The distance all the way round the hexagon (**red path**) = 6.
- The distance all the way round the circle (**green path**) is a little more: in fact, it's **$2\pi r = 6.283\dots$**



Arcs Subtending Angles: the Radian

- It's 360° all the way round the circle, that's 60° from each of the equilateral triangles.
- We say that the **arc** of circle between A and B “**subtends**” an angle of 60° at the center of the circle.
- One radian is defined as the angle subtended by an arc equal in length to the radius of the circle.



Clicker Question

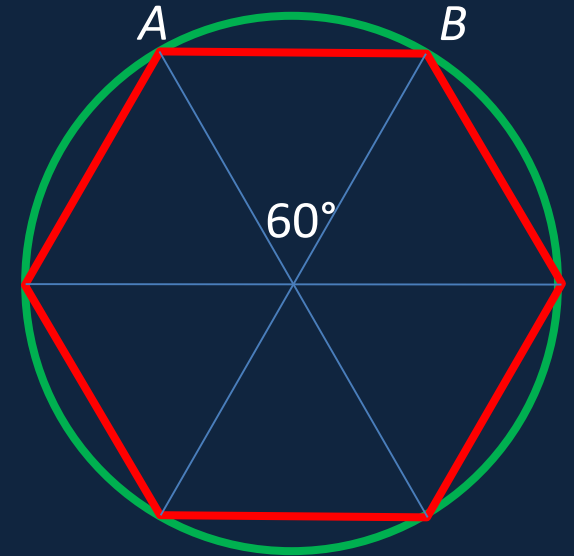
One radian is:

- A. 60°
- B. 120°
- C. A bit less than 60°
- D. A bit more than 60°
- E. None of the above

Clicker Answer

One radian is:

- A. 60°
- B. 120°
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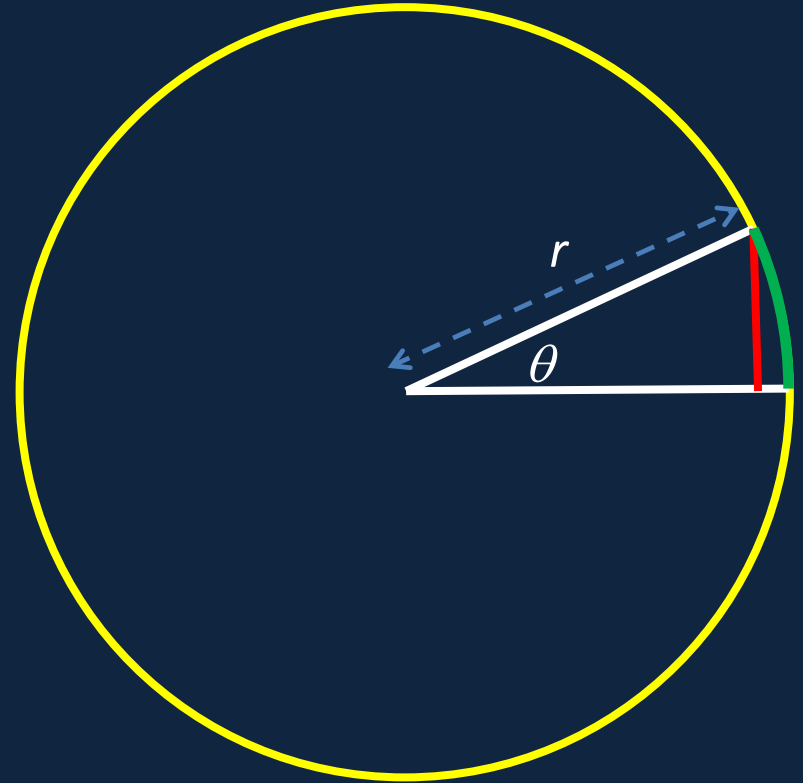
The **straight line** distance from A to B is one side of an equilateral triangle, exactly one radius, the **arc** from A to B is a bit further—so 60° is a little *more* than one radian.

Full Circle

- For a circular path of radius r , if you walk a distance r along the path, you have gone around an angle of one radian relative to the center.
- If you walk all the way around the path, you have of course gone through 360° .
- BUT you've walked a total distance $2\pi r$, and therefore around an angle of 2π radians.
- Conclusion: $360^\circ = 2\pi \text{ radians}$

Radians and Trig

- Measuring the angle θ in radians,
- $\theta = (\text{length green arc})/r$
and
- $\sin \theta = (\text{length red line})/r$
- so for small angles
 $\sin \theta \approx \theta$



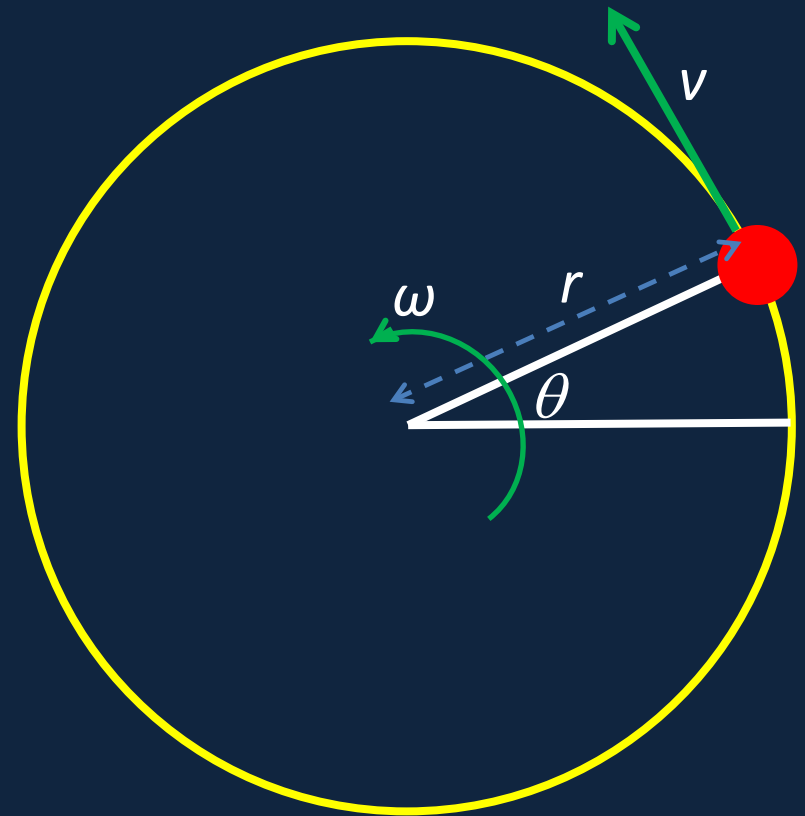
Units for Angular Velocity

- How fast is something rotating?
- Car engine: units **rpm**, revs per minute, **redlines** around 6,000 rpm or 100 revs/sec.
- 1 Hertz, written **1Hz**, means one cycle/sec, used for electrical generators, circuits. (Often called the **rotational frequency**, and written ***f***.)
- Second hand on watch turns at 1 rpm, or $6^\circ/\text{sec}$.
- Earth goes round Sun at very close to $1^\circ/\text{day}$
- (probably why the degree was the original measure of angle.)

Angular Speed and Rim Speed

- If a wheel of radius r rotates one **revolution** per second, a **ball** on the rim is moving at speed $v = 2\pi r$ m/sec.
- If it rotates at one **radian** per sec, $v = r$ m/sec.
- If it rotates at ω rad/sec, $v = \omega r$ m/sec.
- we'll measure angular velocities in radians per second and often use

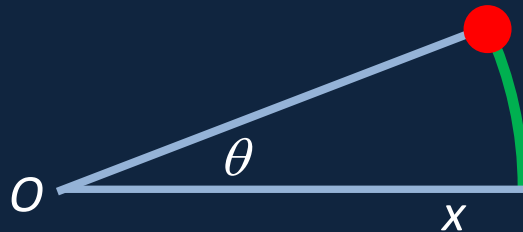
$$v = \omega r$$



Note: $\omega = 2\pi f$, if f is the frequency in cycles per second.

Standard Angular Notation

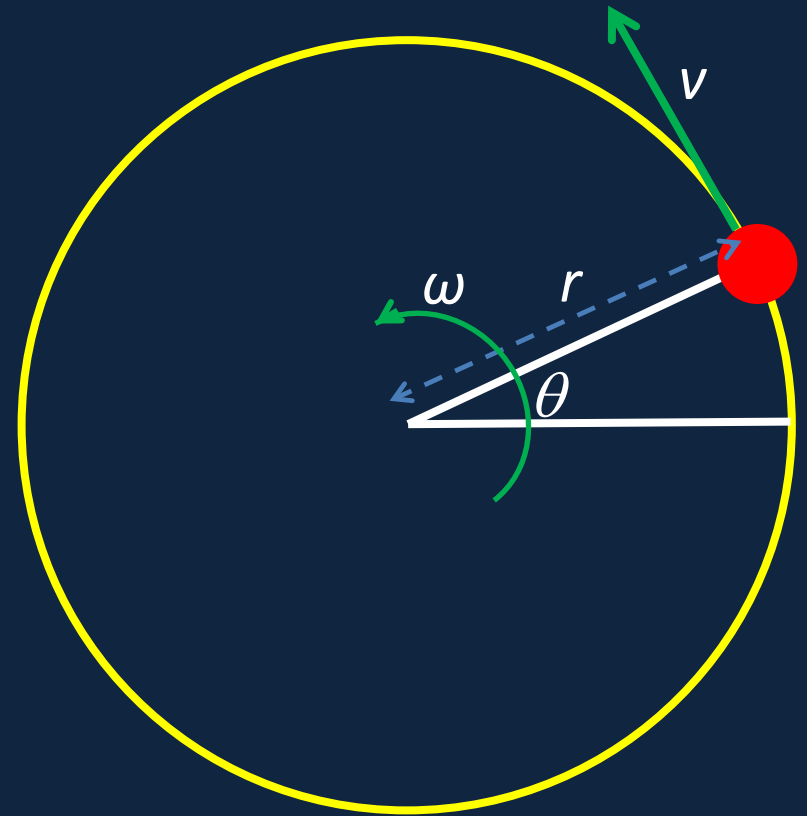
- **Angle:** theta, θ , in radians, measured counterclockwise from the x-axis.



- **Angular velocity:** omega, $\omega = d\theta/dt$.
- **Angular acceleration:** alpha, $\alpha = d\omega/dt = d^2\theta/dt^2$

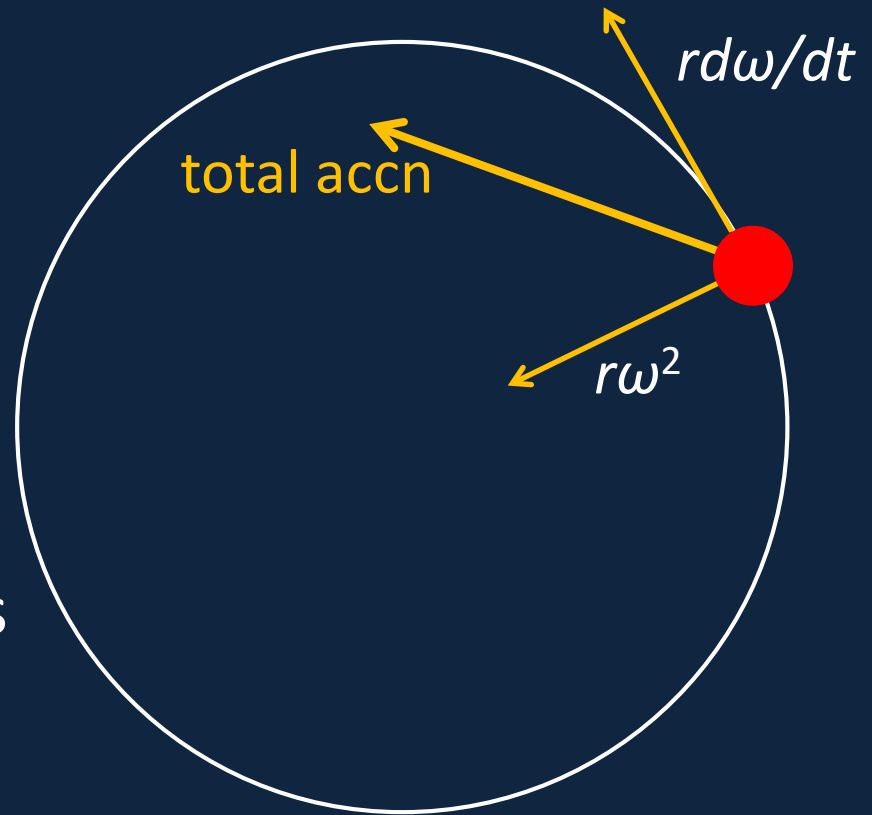
Acceleration

- The tangential speed (along the rim) is $v = r\omega$, so the **tangential acceleration** is
- $a = dv/dt = rd\omega/dt = r\alpha$.
- The **centripetal acceleration** is
 $v^2/r = r\omega^2$.



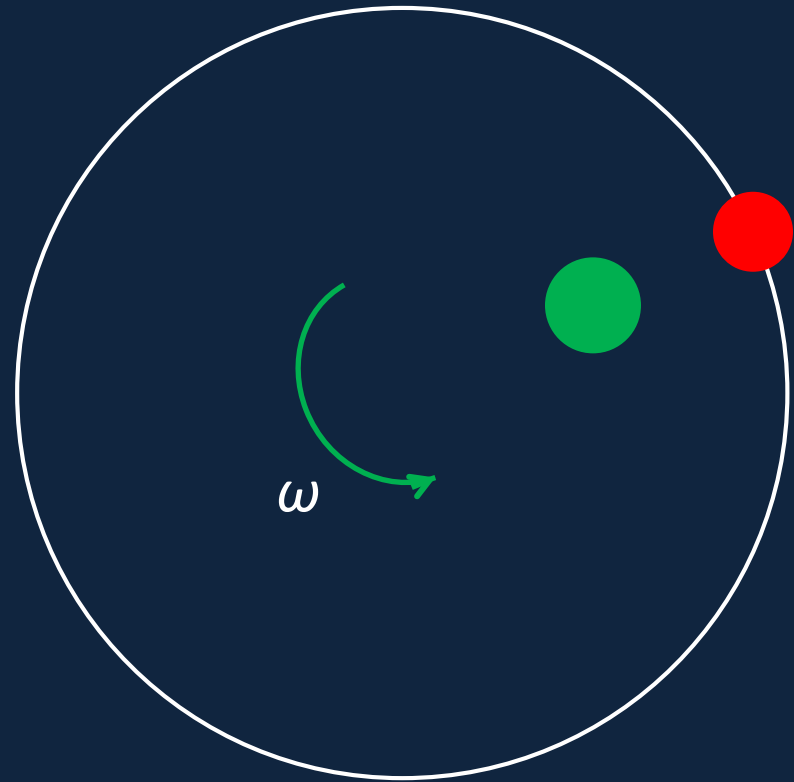
Components of Acceleration

- The tangential speed (along the rim) is $v = r\omega$, so the **tangential acceleration** (parallel to the rim) is $dv/dt = rd\omega/dt = r\alpha$.
- The **centripetal acceleration** is
- $v^2/r = r\omega^2$.
- Note: this formula is useful for comparing accelerations at different radii.



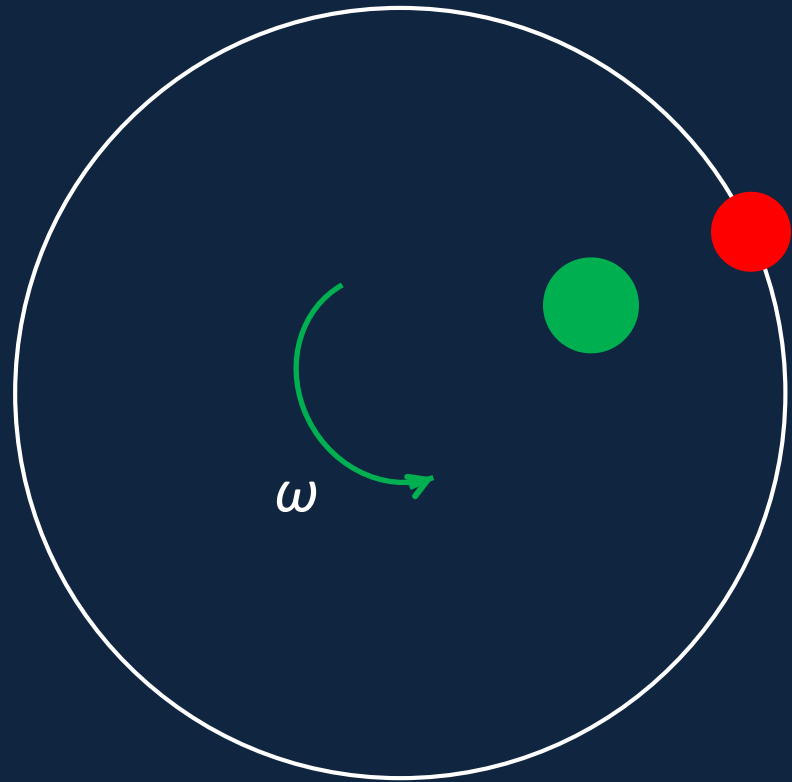
Clicker Question

- A **red** ball and a **green** ball are attached to a wheel as shown. The wheel is rotating at angular velocity ω , with nonzero angular acceleration α .
- Is the **direction of total acceleration** of the **red** ball parallel to that of the **green** ball?
- A Yes. B No.



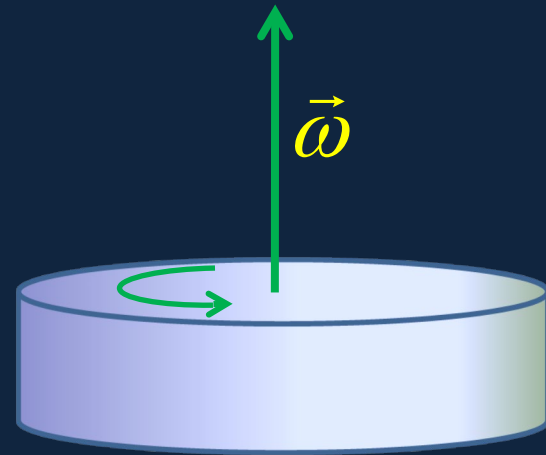
Clicker Answer

- A **red** ball and a **green** ball are attached to a wheel as shown. The wheel is rotating at angular velocity ω , with nonzero angular acceleration α .
- Is the **direction of total acceleration** of the **red** ball parallel to that of the **green** ball?
- A Yes. B No.
- The tangential acceleration of the red ball is $r\alpha$, its centripetal acceleration is $r\omega^2$.
- **The green ball has the same values for the angular variables α and ω** , so if it is at half the radius of the red ball, BOTH components of the acceleration are less by a factor of 2.



Angular Velocity as a Vector

- It will turn out to be essential later to represent angular velocity as a vector, with magnitude equal to the angular speed (radians per second) and direction along the axis of rotation.
- The convention, the “**right hand rule**” is given by curling up your right-hand fingers, your thumb pointing away from the palm, then if the fingers curl in the direction of rotation, the thumb is in the direction of $\vec{\omega}$.



Constant Angular Acceleration

- The formulas for angular velocity and position as functions of time for **constant** angular acceleration are precisely analogous to those for constant linear acceleration derived previously:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

- Just be sure before you use these formulas that you really *do* have **constant** acceleration!

Torque

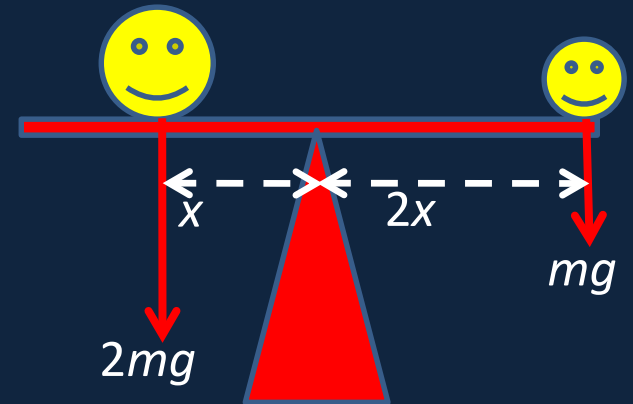
- The two kids shown have the same **torque** about the axle:

- **Torque = force x distance from the axle of the force's line of action.**

- Notation: torque

$$\tau = Fd = 2mgx$$

- Torque is also called “moment of a force” the distance d the “moment arm”.

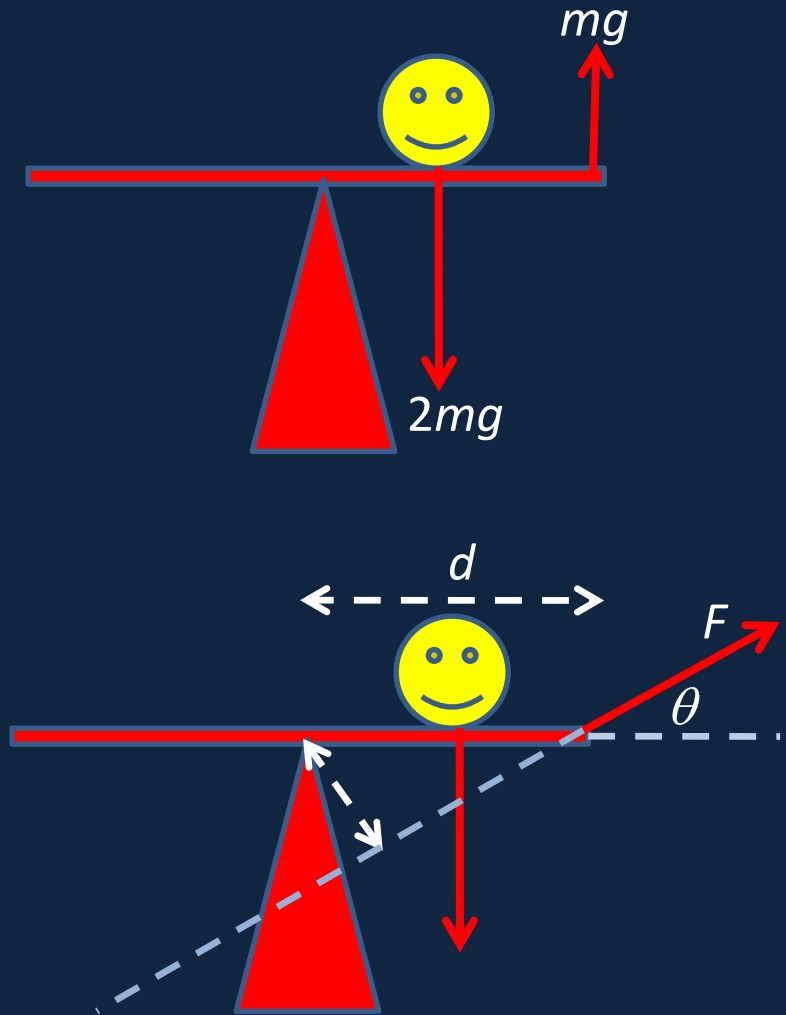


More Ways to Balance Torques...

- The two forces can act on the same side of the axle.
- The force does not need to be perpendicular to the lever arm: BUT only its component perpendicular to the arm exerts torque

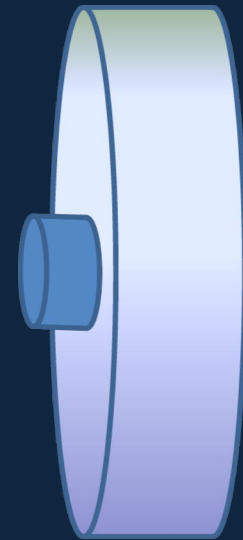
$$\tau = Fd \sin \theta$$

- Alternatively, one can draw the whole line of action of the force and find the perpendicular distance.



Rotational Dynamics

- **Newton's First Law:** a rotating body will continue to rotate at constant angular velocity as long as there is no torque acting on it.
- Picture a grindstone on a smooth axle.
- BUT the axle must be *exactly* at the center of gravity—otherwise gravity will provide a torque, and the rotation will not be at constant velocity!



How is *Angular* Acceleration Related to Torque?

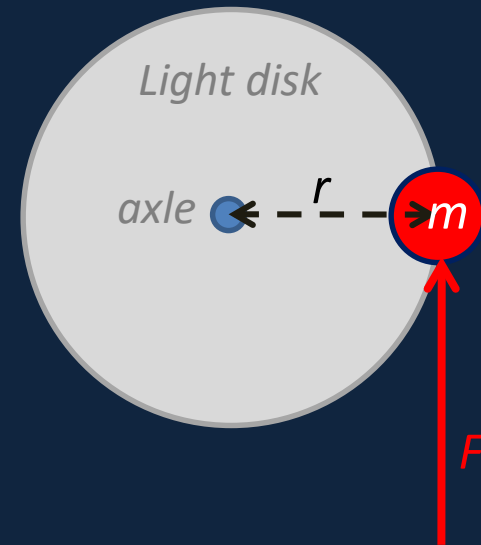
- Think about a tangential force F applied to a mass m attached to a light disk which can rotate about a fixed axis. (A *radially* directed force has zero torque, so does nothing.)

- The relevant equations are:

$$F = ma, a = r\alpha, \tau = rF.$$

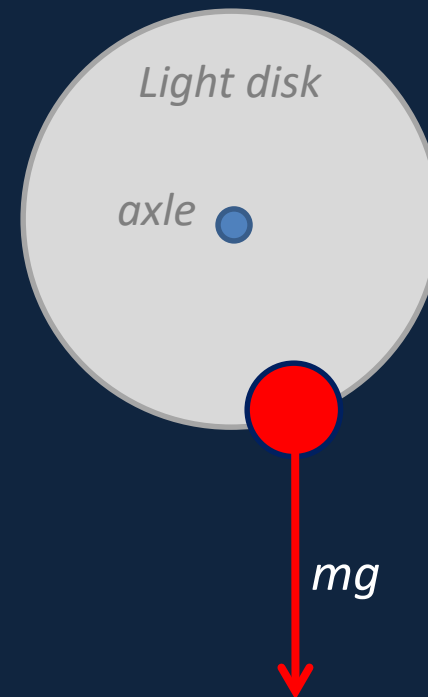
- Therefore $F = ma$ becomes

$$\tau = mr^2\alpha$$



Kinds of Equilibrium

- Suppose now the light disk is in a vertical plane, free to rotate about a horizontal axis.
- If the **red mass** is **at rest at the lowest point**, and is then displaced slightly, the torque from the gravitational force mg will pull it back towards the center. This is called **stable equilibrium**.
- The **red mass** can be **at rest at the topmost point**—but this is **unstable equilibrium**.
- If $g = 0$, we have **neutral equilibrium**.

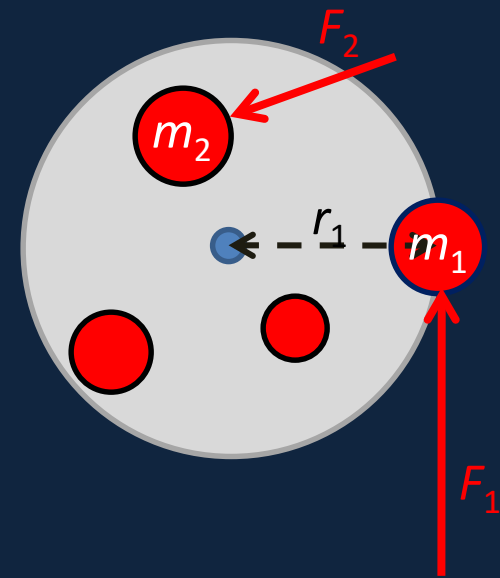


Newton's Second Law for Rotations

- For the **special case** of a mass m constrained by a light disk to circle around an axle, the angular acceleration α is proportional to the torque τ **exactly** as in the linear case the acceleration a is proportional to the force F .
- The angular equivalent of inertial mass m is the **moment of inertia** mr^2 .

More Complicated Rotating Bodies

- Suppose now a light disk has several different masses attached at different places, and various forces act on them. As before, radial components cause no rotation, we have a sum of torques.
- **BUT the rigid disk will cause a force on one mass to cause a torque on all the others! How do we handle *that*?**



Newton's Third Law for a Rigid Rotating Body

- If a rigid body is made up of many masses m_i connected by rigid rods, the force exerted along the rod of m_i on m_j is equal in magnitude and opposite in direction to that of m_j on m_i , therefore **the internal torques come in equal and opposite pairs, and therefore cannot contribute to the angular acceleration.**
- It follows that the angular acceleration is generated by the sum of the **external** torques.

Moment of Inertia of a Solid Body

- Consider a flat square plate rotating about a perpendicular axis with angular acceleration α . One small part of it, Δm_i , distance r_i from the axle, has equation of motion

$$\tau_i = \tau_i^{\text{ext}} + \tau_i^{\text{int}} = \Delta m_i r_i^2 \alpha$$

- Adding contributions from all parts of the wheel

$$\tau = \sum_i \tau_i^{\text{ext}} = \left(\sum_i \Delta m_i r_i^2 \right) \alpha = I \alpha$$

- I is the **Moment of Inertia**.

