

Temperature, Expansion, Ideal Gas Law

Physics 1425 Lecture 30

Everything's Made of Atoms

- This idea was only fully accepted about 100 years ago—in part because of Einstein's analysis of **Brownian motion**.
- Brown, who studied the sex life of plants, noticed a lot of jiggling pollen grains under his microscope in 1827. He assumed it was because they were alive, but later found the identical jiggling with definitely dead stone powder.
- This was not understood for half a century....!

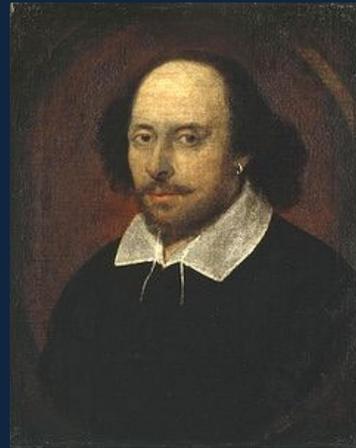
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Size and Mass of Atoms

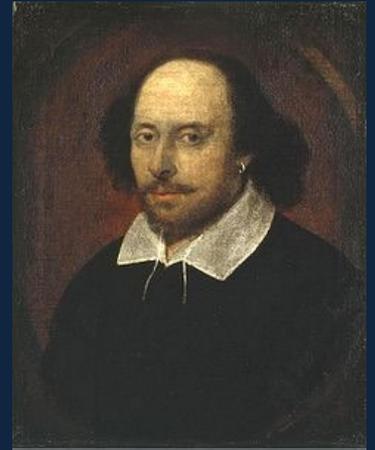
- The hydrogen atom is about 10^{-10} m across, others are a few times bigger.
- Avogadro's Number: $N_A = 6.02 \times 10^{23}$, the number of atoms (or molecules) in one gram-mole, 22.4 L volume at NTP.
- The atomic mass unit is 1.66×10^{-27} kg. The mass of a molecule in amu = mass of N_A atoms in grams: one gram mole of H_2O is 18 grams.
- **NOTE:** this is just a reminder—you should be very familiar with all this from chemistry!

Clicker Question

- Assume the molecules Shakespeare breathed out in his last breath (say, one liter) are now uniformly distributed throughout the atmosphere. What is the probability you breathed one in just now, in your most recent breath?
- A. 1 in 10,000
 - B. 1 in 1,000
 - C. 1 in 100
 - D. 1 in 10
 - E. More likely than not.



Clicker Answer



The answer is: more likely than not.

There are 6×10^{23} molecules in 22.4 L, so about 3×10^{22} in one liter.

The Earth's atmosphere has volume $4\pi R^2 d$, take $R = 6 \times 10^6$ m, $d = 2 \times 10^4$ m. This gives a volume about 10^{15} m³, or 10^{21} L.

Measuring Temperature

- We can tell by touch if something is hot or cold, but this is unreliable. The first serious attempt to measure temperature was by Galileo in 1597.
- A glass bulb has a long thin neck, the end of which is immersed in liquid.
- As the temperature varies, the gas in the bulb changes volume, sucking up liquid or pushing it down.

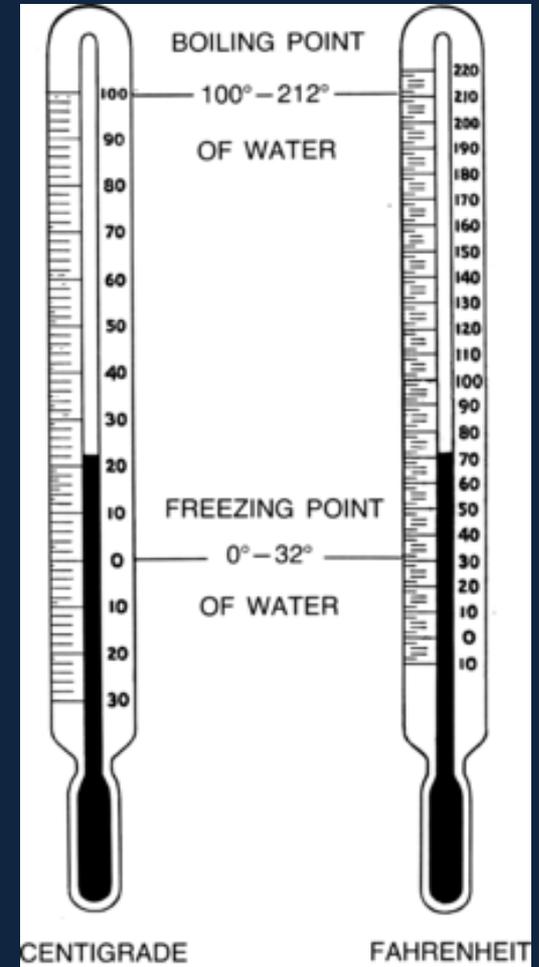


Clicker Question

- Why was Galileo's thermometer no good for comparing temperatures from day to day?
 - A. The fluid would evaporate.
 - B. The gas expansion was too small to see clearly.
 - C. This instrument is also a barometer.

Thermometers

- Many thermometers use the **expansion of a liquid** as a measure of temperature.
- **You should be familiar** with the two standard temperature scales and how to convert between them.
- **Bimetallic strips**, two metals with differing expansion rates welded together, bend when heated, and make very robust thermometers.



Zeroth Law of Thermodynamics

- If two things at different temperatures are in thermal contact, so heat can flow, and no heat is being supplied from or being drained to the environment, **they will reach the same temperature**. They are then said to be in “thermal equilibrium”.
- **The Zeroth Law states that if A is in thermal equilibrium with B , and B is with C , then A will be with C .**
- If this wasn't true, thermometers would be *meaningless*—but thermodynamics guys like to see it written down...

Thermal Expansion

- A solid rod will increase in length when heated, typically by of order 10^{-5} of its original length for each degree celsius (centigrade). (It will expand by the same proportion in the other directions too.)
- This means there will only be a change of one part in 1,000 over a 100°C temperature range, so these changes are not visible to the naked eye, some device is needed to detect them.

Thermal Expansion Notation

- The coefficient of linear expansion, denoted by α , is defined by $\Delta\ell/\ell_0 = \alpha\Delta T$.
- $\alpha = 1.2 \times 10^{-5}$ for iron, 0.9×10^{-5} for glass.
- The coefficient of volume expansion β is defined by $\Delta V/V_0 = \beta\Delta T$.

Relating Linear and Volume Expansion

- Imagine a cube of solid of side L , volume $V = L^3$.
- On increasing the temperature by ΔT , the length of each side goes from L to $L(1 + \alpha\Delta T)$, so the volume increases from L^3 to $L^3(1 + \alpha\Delta T)^3$.

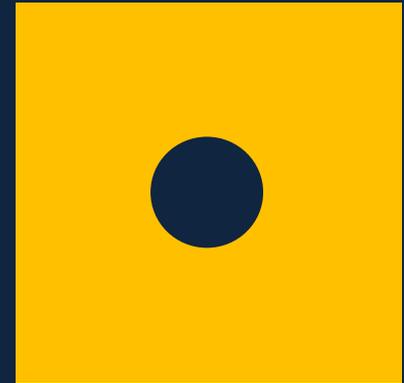
- Now $\alpha\Delta T$ is a very small number, so

$$\begin{aligned}L^3(1 + \alpha\Delta T)^3 &= L^3(1 + 3\alpha\Delta T + 3(\alpha\Delta T)^2 + (\alpha\Delta T)^3) \\ &= L^3(1 + 3\alpha\Delta T) - \text{those other terms are} \\ &\text{really tiny! (for solids, } \alpha \sim 10^{-5}\text{)}\end{aligned}$$

- Recall the volume goes to $V(1 + \beta\Delta T)$: so $\beta = 3\alpha$

Clicker Question

- I have a **square brass plate with a hole in it**. I put it in the oven until it reaches a **high uniform temperature**, then immediately **measure the hole** very accurately.
- What do I find?
 - A. The hole is bigger than it was before heating.
 - B. The hole is smaller.
 - C. It's the same size.



Clicker Question

- Coefficient of linear expansion of aluminum: 2.5×10^{-5} .
- An aluminum plate $2\text{cm} \times 3\text{cm}$ is heated through 10°C .
By how much does its **area** increase?

- A. $1.5 \times 10^{-3} \text{ cm}^2$.
- B. $3.0 \times 10^{-3} \text{ cm}^2$.
- C. $4.5 \times 10^{-3} \text{ cm}^2$.
- D. $6.0 \times 10^{-3} \text{ cm}^2$.

Clicker Question

- Coefficient of linear expansion of aluminum: 2.5×10^{-5} .
- A solid aluminum sphere is heated through 1°C .
What is its fractional change in **density**?

- A. $2.5 \times 10^{-5} \text{ cm}^2$.
- B. $5.0 \times 10^{-5} \text{ cm}^2$.
- C. $7.5 \times 10^{-5} \text{ cm}^2$.
- D. $-7.5 \times 10^{-5} \text{ cm}^2$.
- E. $-2.5 \times 10^{-5} \text{ cm}^2$.

Volume Expansion Coefficients for Liquids

- Most **common organic liquids have coefficients around 10^{-3}** : *much* greater than solids, and a good reason for not filling a gas tank to the very top!
- But mercury has a low expansion coefficient for a liquid: 1.8×10^{-4} . (Still 20 times that for glass—mercury thermometers work fine.)

Water, unlike almost all other liquids, expands when cooling from 4°C to freezing. It also has a highly variable coefficient of expansion over its whole temperature range:

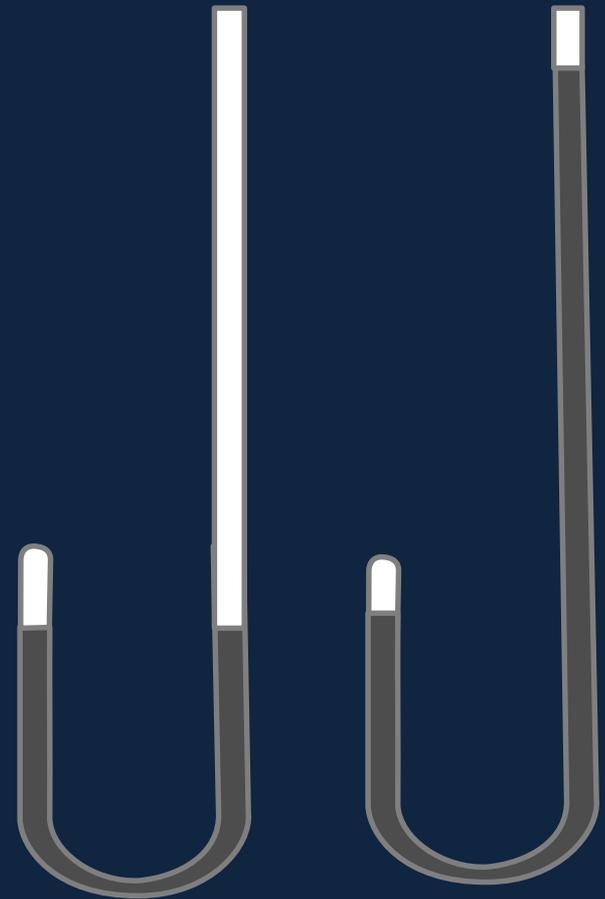
°C range	β (units 10^{-3}) H ₂ O
10-20	0.15
20-30	0.25
30-40	0.35
40-60	0.46
60-80	0.59
80-100	0.70

Clicker Question

- A manufacturer of mercury in glass thermometers decides to **upgrade his product to Pyrex** glass, which holds up better to high heat because it has a lower coefficient of expansion. He makes the new thermometers **identical** to the old in all dimensions.
- To check his product, he puts a new one and an old one together in water, and heats the water slowly. What does he see?
 - A. The mercury in the new one rises faster.
 - B. The mercury in the old one rises faster.
 - C. They both rise at the same rate.

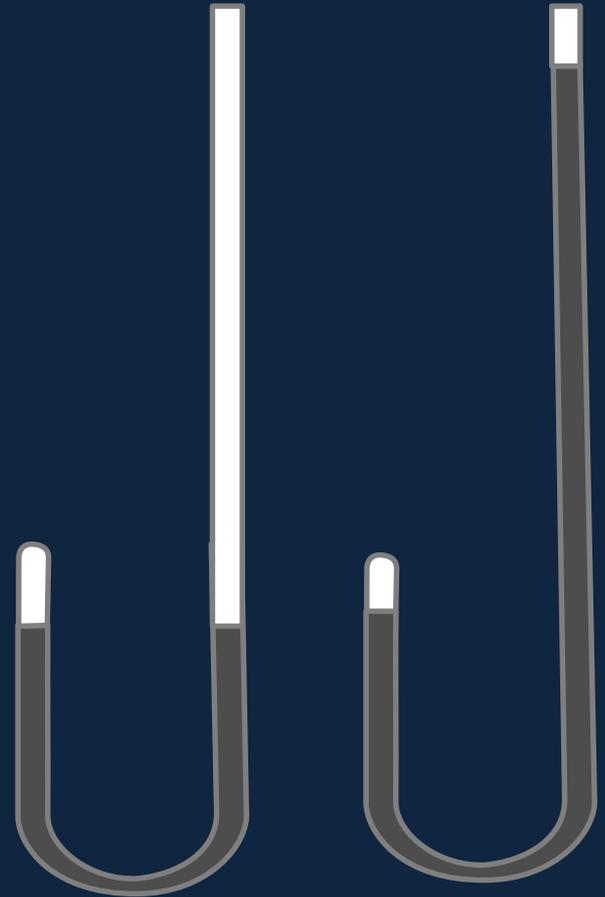
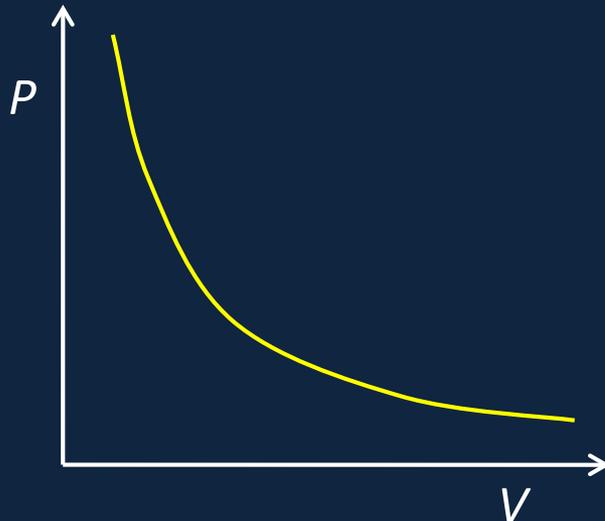
Boyle's Law

- Boyle (born in 1627, the 14th child of the Earl of Cork) discovered his Law himself. He used a U-shaped glass tube, closed at one end, open at the other. He first carefully poured in mercury, with the tube almost horizontal so the **trapped air was at atmospheric pressure**: the levels in the two arms were equal.
- He then poured in **thirty inches more of mercury**, that's **one atmosphere**, and found the **trapped air now had half the original volume**.



Boyle's Law

- Boyle knew that fast compression heats a gas, so before measuring the volume at the higher pressure, he waited for the air to cool back down. He repeated the experiment at different added pressures.
- He found that **at constant temperature T , pressure \times volume = constant.**



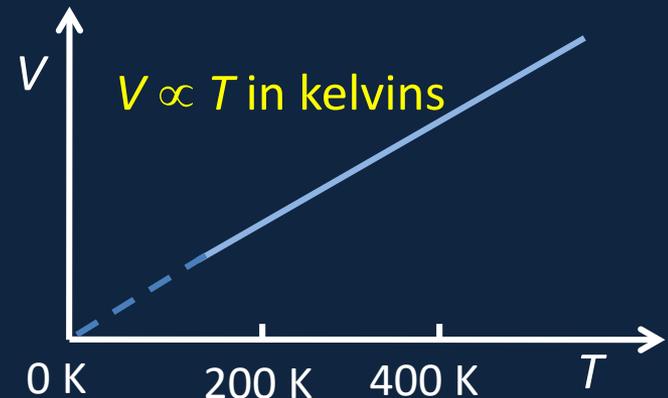
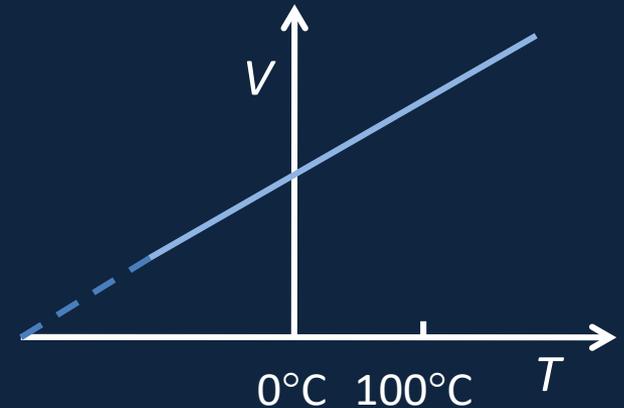
Expansion Coefficient for Gases

- Charles's Law

- A century after Boyle, Charles discovered that **at constant pressure**, and far from liquefaction, **all gases have the same expansion coefficient**, in fact

$$V(T \text{ } ^\circ\text{C}) = V(0 \text{ } ^\circ\text{C})(1 + T/273)$$

- This suggests the gas **volume shrinks to zero at -273** , but of course it liquefies first.
- The Kelvin temperature scale:
- $T \text{ (K)} = T \text{ (}^\circ\text{C)} + 273.15$



The Ideal Gas Law

- We can combine Boyle's law and Charles's law to find $PV \propto T$ for gases well away from liquefaction. Note this also implies that at constant volume, $P \propto T$.
- The standard notation for this Ideal Gas Law is:

$$PV = nRT$$

for n moles of gas, $R = 8.314 \text{ J}/(\text{mol}\cdot\text{K})$ is the universal gas constant.

Gas Law Exercise

- 100.0 L of oxygen at 27.0°C and absolute pressure 10 atm are compressed to 50.0 L. The gas is subsequently taken to 177.0°C. What is the final pressure?
- $P_1V_1/T_1 = P_2V_2/T_2$: so $P_2 = P_1(V_1/V_2)(T_2/T_1)$
 $P_2 = 10 \times (100/50) \times (450/300) = 30 \text{ atm.}$

On homework: (1) watch out for gauge pressure and absolute pressure! (2) the gas law always has T in kelvins. (3) for given volume of gas and temperature, the pressure is determined by the total number of molecules (which could be single atoms, for example He) and not by the masses of the atoms. 10^{22} He atoms will exert the same pressure as 10^{22} oxygen molecules at given T, V .