

Capacitors

Physics 2415 Lecture 8

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Today's Topics

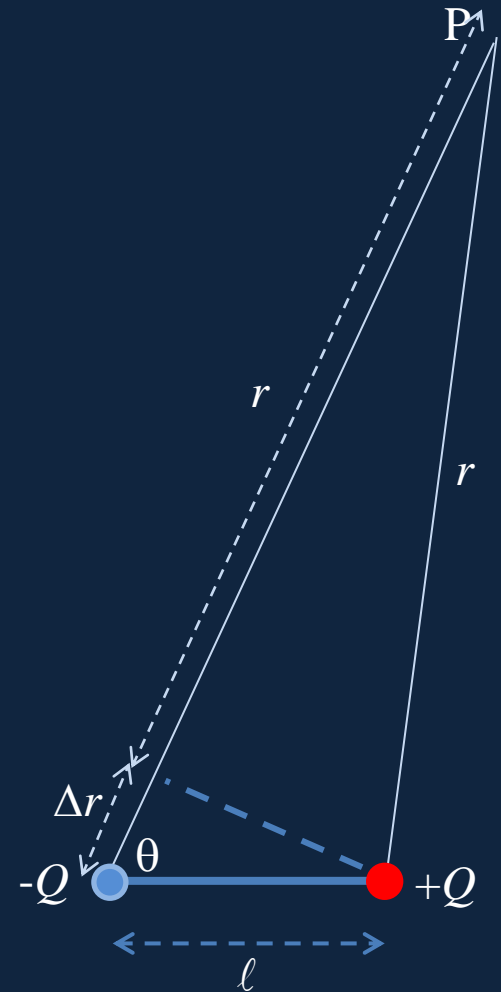
- Dipole Potential, Lightning Conductors
- Storing Charge on a Spherical Conductor
- Parallel Plate Capacitors
- Cylindrical Capacitors
- Capacitors in Series and Parallel

Dipole Potential Far Away

- At distances $r \gg \ell$, the charge separation distance, the dipole potential at a point P has a simple form:

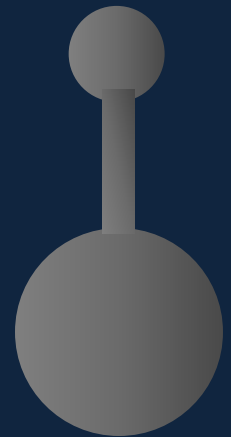
$$V(\mathbf{P}) = \frac{kQ}{r} - \frac{kQ}{r + \Delta r} = \frac{kQ\Delta r}{r(r + \Delta r)}$$
$$\cong \frac{kQ\ell \cos \theta}{r^2} \cong \frac{kp \cos \theta}{r^2}$$

- Recall the dipole moment $p = Q\ell$, and we've approximated for $r \gg \Delta r$.



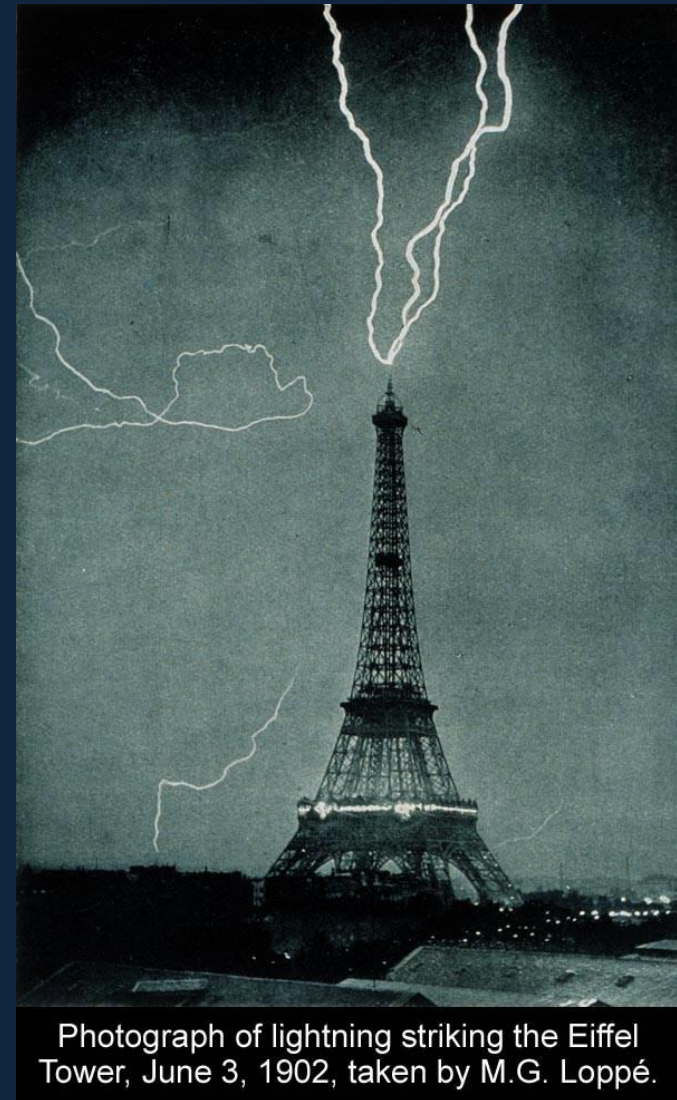
Connected Spherical Conductors

- Two spherical conductors are connected by a conducting rod, then charged—all will be at the same potential.
- **Where is the electric field strongest?**
 - A. At the surface of the small sphere.
- Take the big sphere to have radius R_1 and charge Q_1 , the small R_2 and Q_2 .
- Equal potentials means $Q_1/R_1 = Q_2/R_2$.
- Since $R_1 > R_2$, field $kQ_1/R_1^2 < kQ_2/R_2^2$.
- This means the **surface charge density is greater on the smaller sphere!**



Electric Breakdown of Air

- Air contains free electrons, from molecules ionized by cosmic rays or natural radioactivity.
- In a strong electric field, these electrons will accelerate, then collide with molecules. If they pick up enough KE between collisions to ionize a molecule, there is a “chain reaction” with rapid current buildup.
- This happens for E about $3 \times 10^6 \text{V/m}$.



Voltage Needed for Electric Breakdown

- Suppose we have a sphere of radius 10cm, 0.1m.
- If the field at its surface is just sufficient for breakdown,

$$3 \times 10^6 = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2}$$

- The voltage

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R} = 3 \times 10^6 R = 300,000V$$

- For a sphere of radius 1mm, 3,000V is enough—there is discharge before much charge builds up.
- This is why lightning conductors are pointed!

Charged Sphere Potential and Field

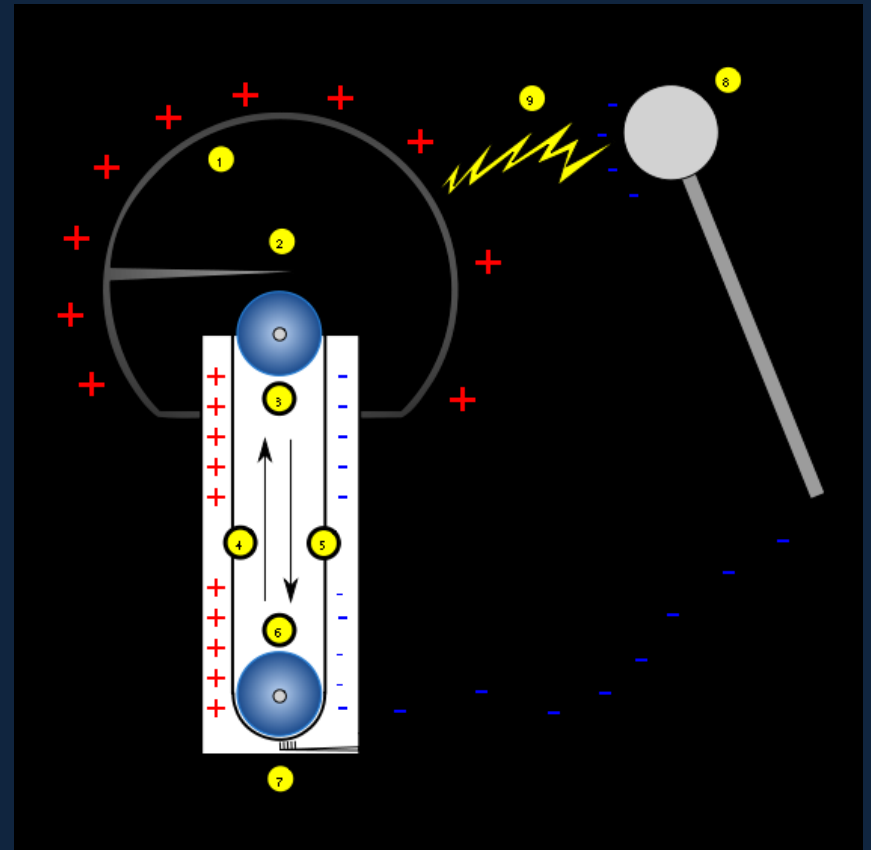
- For a spherical conductor of radius R with total charge Q uniformly distributed over its surface, we know that

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q\hat{r}}{r^2} \quad \text{and} \quad V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}.$$

- The field at the surface is related to the surface charge density σ by $E = \sigma/\epsilon_0$.
- Note this checks with $Q = 4\pi R^2 \sigma$.

Storing Charge on a Sphere

- In the van der Graaff machine, electric charge is carried up to a sphere on an insulating belt, the belt drive overcoming the electrostatic repulsion.
- As charge builds up, so does the voltage—to the point where breakdown of air or part of the insulator occurs.



Capacitance of a Sphere

- Recall that the potential (voltage) at the surface of the sphere is

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$$

for a sphere of radius R .

- Notice that the stored charge Q is **directly proportional** to the voltage V , and the ratio of the two is called the **capacitance**:

$$C = Q/V$$

Unit Capacitance

- Writing $C = Q/V$ defines the unit capacitance: adding one coulomb of charge must raise the potential of a capacitance with $C = 1$ by one volt.
- This unit capacitance is called one farad, 1F, in honor of Michael Faraday, the first person to understand electric and magnetic fields.
- For the spherical capacitor,

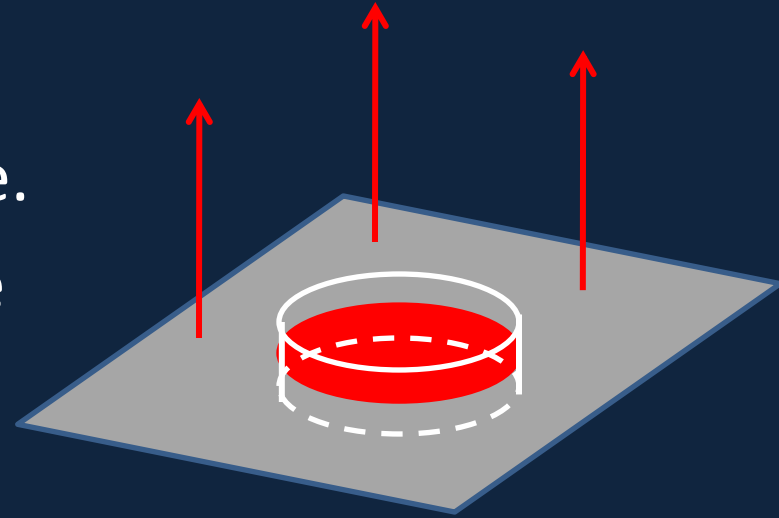
$$Q/V = 4\pi\epsilon_0 R = R / (9 \times 10^9)$$

so a 1F sphere capacitor is bigger than the Sun!

Charge on Surface of a Conductor

- For a flat conducting surface, the electric field is perpendicularly outward, or a current would arise.
- We have a sheet of charge on the surface, so we take the same Gaussian pillbox as for the sheet of charge, but this time **there is no electric field pointing downwards into the conductor.**
- Therefore Gauss' Law gives

$$AE = \frac{\sigma A}{\epsilon_0}, \text{ so } E = \frac{\sigma}{\epsilon_0}$$

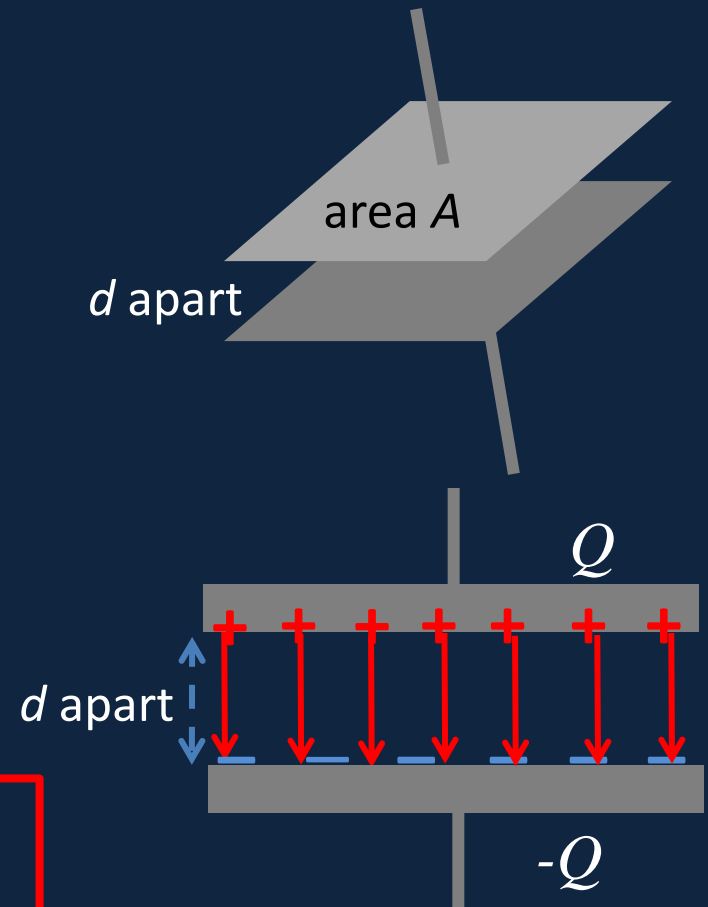


Parallel Plate Capacitor

- Almost all capacitors are **parallel plate capacitors**: two conducting plates each of area A a constant distance d apart.
- For total charge Q on the top plate and $-Q$ on the bottom, taking $d \ll \sqrt{A}$,
- $E = \sigma/\epsilon_0 = Q/A\epsilon_0$ and $V = Ed$,

so

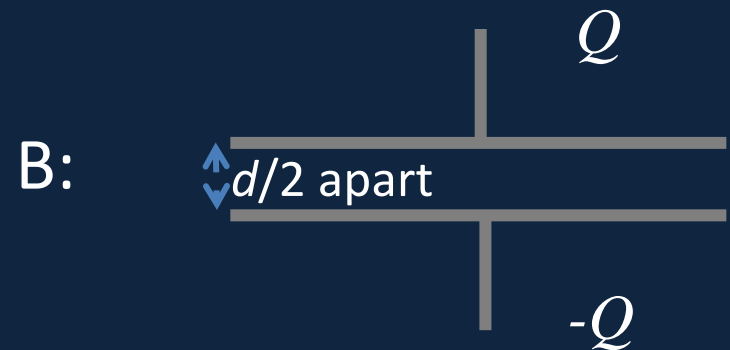
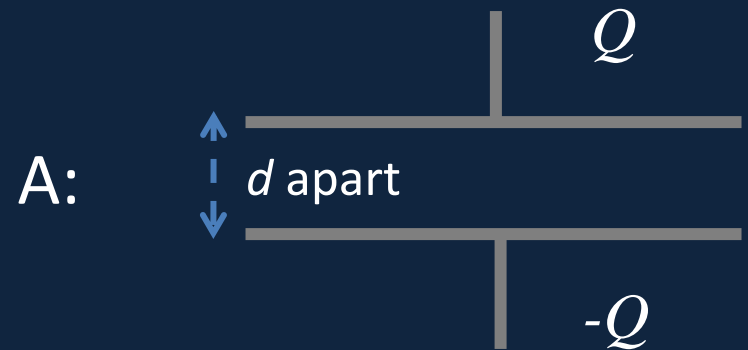
$$V = \frac{Qd}{A\epsilon_0} = \frac{Q}{C} \text{ where } C = \epsilon_0 \frac{A}{d}$$



Charge will settle on *inside* surfaces

Clicker Question

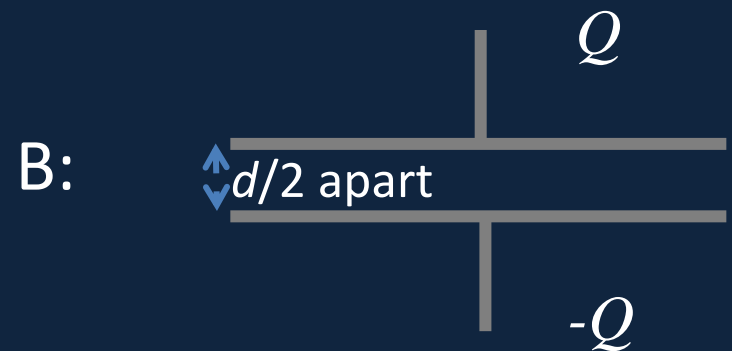
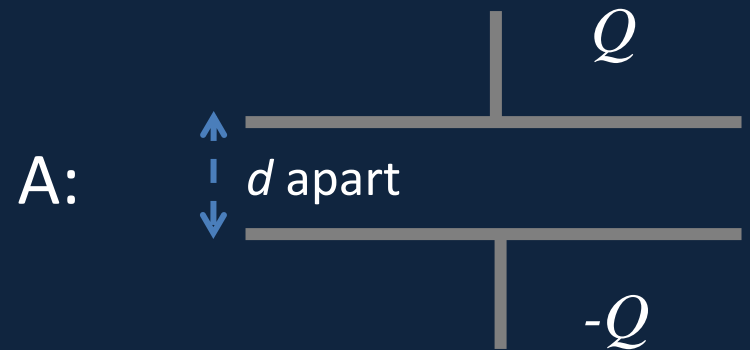
- The two capacitors on the right have the **same size plates**, and hold the **same charge Q** .
- For which capacitor is the voltage V between the plates greater?



C: they have the same V

Clicker Answer

- The two capacitors on the right have the **same size plates**, and hold the **same charge Q** .
- The **same charge density** means the **same strength electric field** between the plates: so to move a charge between plates for A takes **twice the work**—**twice the voltage**.



C: they have the same V

Clicker Question

- If the voltage applied to a parallel plate capacitor is doubled, what happens to the capacitance C ?
 - A. It's doubled
 - B. It's halved
 - C. It doesn't change

Clicker Answer

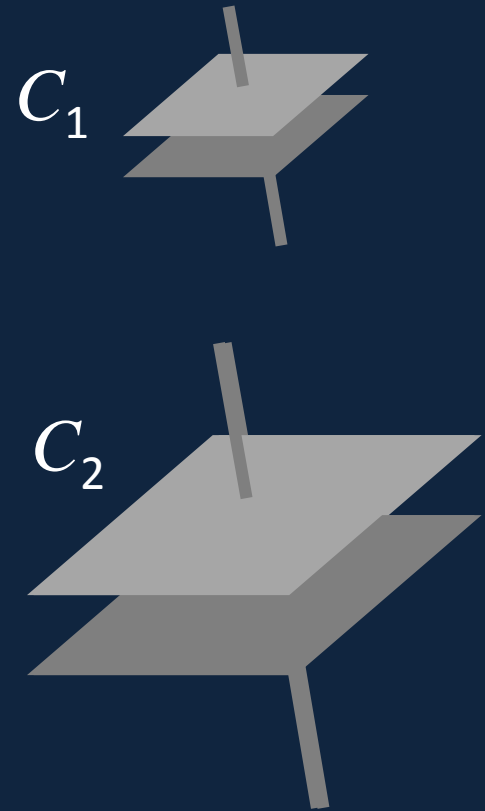
- If the voltage applied to a parallel plate capacitor is doubled, what happens to the capacitance C ?

It doesn't change: capacitance depends on plate area and plate separation, NOT on the charge stored.

Don't use formulas before thinking, however briefly!

Clicker Question

- Capacitor C_2 is a scale model of capacitor C_1 , with all linear dimensions up by a factor of 2.
- What is the ratio of capacitances?
 - A. C_2 is 8 times C_1
 - B. C_2 is 4 times C_1
 - C. C_2 is twice C_1

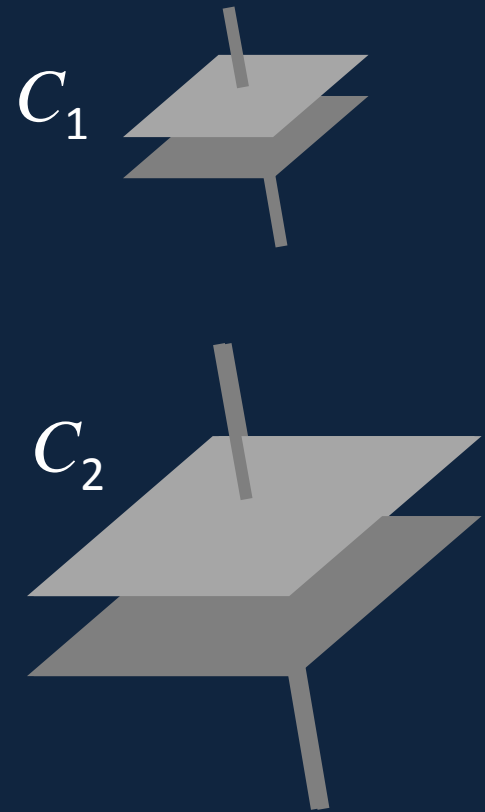


Clicker Answer

- Capacitor C_2 is a scale model of capacitor C_1 , with all linear dimensions up by a factor of 2.
- What is the ratio of capacitances?

C_2 is twice C_1 :

Area is up by a factor of 4, but doubling of separation distance halves capacitance.



Clicker Question

- A parallel plate capacitor, consisting simply of two metal plates held parallel, is charged with a battery to a voltage V , then the battery is disconnected.
- The plates are now physically pulled further apart : what happens to the voltage?
 - A. It stays the same
 - B. It decreases
 - C. It increases

Clicker Answer

- A parallel plate capacitor, consisting simply of two metal plates held parallel, is charged with a battery to a voltage V , then the battery is disconnected.
- The plates are now physically pulled further apart : what happens to the voltage?
- It increases: the charge on the plates cannot change, so the electric field stays the same, and voltage is field strength x distance.

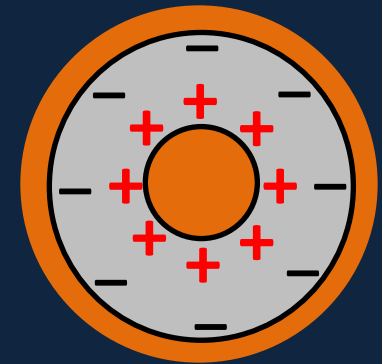
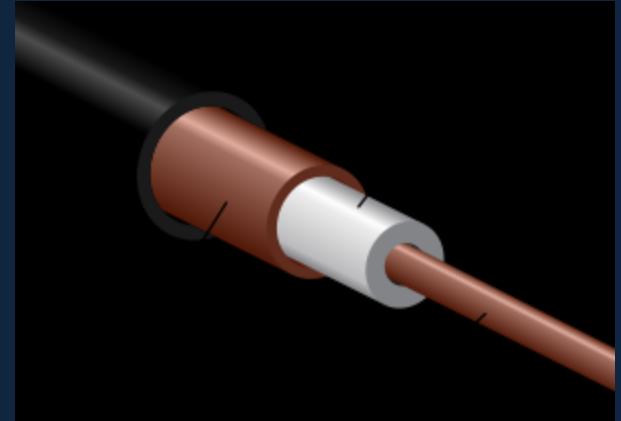
Cylindrical Capacitor

- A coaxial cable is a cylindrical capacitor.
- For charge density λ C/m on the inside wire (and so $-\lambda$ on the inside of the outer cylinder) the radial field $E = \lambda/2\pi\epsilon_0 r$ and

$$V = \int_{R_1}^{R_2} E(r) dr = \frac{\lambda}{2\pi\epsilon_0} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{R_2}{R_1}$$

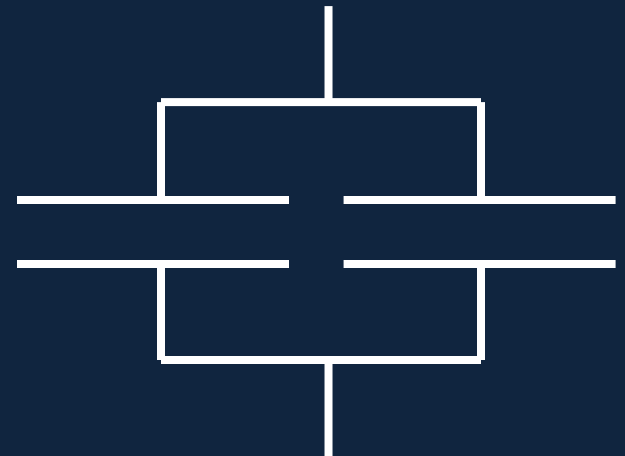
- SO

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_0 \ell}{\ln(R_2/R_1)} \text{ for length } \ell$$

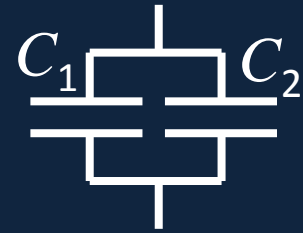


Capacitors in Parallel

- Let's look first at hooking up two identical parallel plate capacitors in parallel: that means the **wires from the two top plates are joined**, similarly at the bottom, so effectively they become one capacitor.
- **What is its capacitance?** From the picture, the combined capacitor has twice the area of plates, the same distance apart.
- We see that $C = C_1 + C_2$



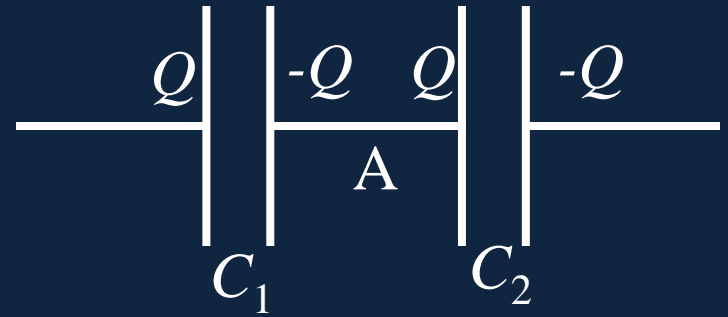
Capacitors in Parallel



- If two capacitors C_1 , C_2 are wired together as shown they have the **same voltage V** between plates.
- Hence they hold charges $Q_1 = C_1V$, $Q_2 = C_2V$, for total charge $Q = Q_1 + Q_2 = (C_1 + C_2)V = CV$.
- So **capacitors in parallel just add:**

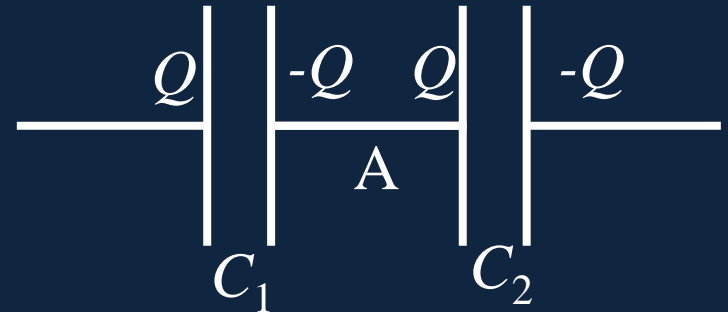
$$C = C_1 + C_2 + C_3 + \dots$$

Capacitors in Series



- Regarding the above as a single capacitor, the important thing to realize is that in adding charge via the outside end wires, **no charge is added to the central section labeled A** : it's isolated by the gaps between the plates.
- Charge **Q on the outside plate of C_1 will attract $-Q$** to the other plate, this has to come from C_2 , as shown.
- **Series capacitors all hold the same charge.**

Capacitors in Series



- Series capacitors all hold the same charge.
- The voltage drop V_1 across C_1 is $V_1 = Q/C_1$.
- The voltage drop across C_2 is $V_2 = Q/C_2$.
- Denoting the total capacitance of the two taken together as C , then the total voltage drop is $V = Q/C$.
- But $V = V_1 + V_2$, so $Q/C = Q/C_1 + Q/C_2$,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$