Electric Currents and Resistance II

Physics 2415 Lecture 11

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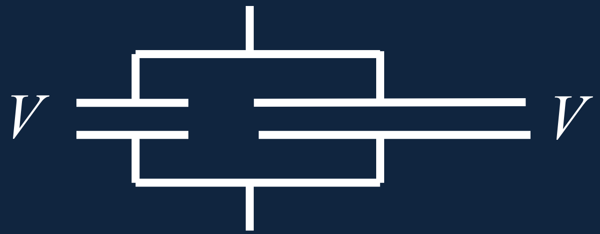
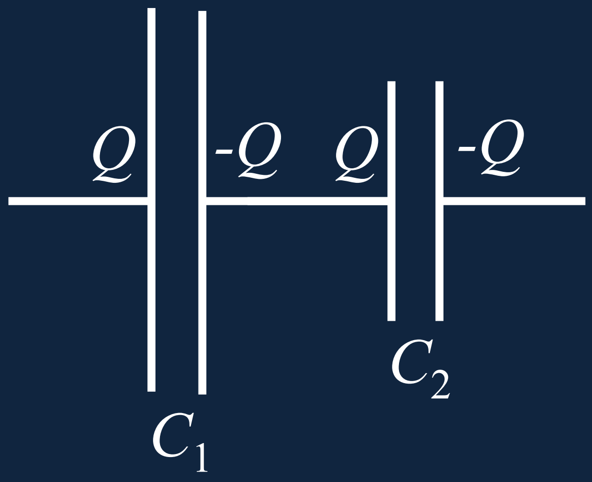
Today's Topics

- First we'll mention capacitors
- Power usage: kWh, etc.
- The microscopic picture
- Temperature dependence of resistivity
- Drift speed and electron speed
- AC and DC
- Semiconductors and superconductors

Know This…

- Capacitors in parallel (any number) are all at the same voltage V.
- Capacitors in series (any number) all carry the same charge Q.
- Putting these facts together with $V = Q/C$ can solve a lot of problems!





Resistance and Resistivity

- To summarize: for a given material (say, copper) the resistance of a piece of uniform wire is proportional to its length ℓ and inversely proportional to its cross-sectional area A.
- This is written: $$R = \rho \frac{\ell}{A}$$

  where ρ is the resistivity.
- For copper, $\rho = 1.68 \times 10^{-8}\ \Omega \cdot \text{m}.$

Electric Power

- Remember voltage is a measure of potential energy of electric charge, and if one coulomb drops through a potential difference of one volt it loses one joule of potential energy.
- So a current of I amps flowing through a wire with V volts potential difference between the ends is losing IV joules per sec.
- This energy appears as heat in the wire: the electric field accelerates the electrons, which then bump into impurities and defects in the wire, and are slowed down to begin accelerating again, like a sloping pinball machine.

Power and Energy Usage

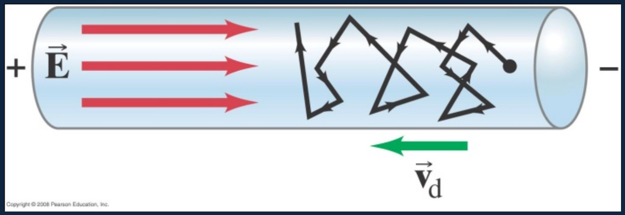
- Using Ohm's law, we can write the power use of a resistive heater (or equivalent device, such as a bulb) in different ways:

$$P = IV = I^2 R = V^2 / R$$

- The unit is watts, meaning joules per second.
- Electric meters measure total energy usage: adding up how much power is drawn for how long, the standard unit is the kilowatt hour:
- 1 kWh = 1,000x3,600J = 3.6MJ

Microscopic Picture of Conductivity

- The total current down the wire is I; if we assume it's uniform over the cross section area A (which it is) there is a current density $j = I/A$. (units: amps/m^2)
- A constant E field gives a steady current. This means the electrons are bouncing off things, like a sloping pinball machine, otherwise the current would keep accelerating.



What are the Electrons Bouncing off?

- Not the atoms! It's found experimentally that electrons pass dozens or often hundreds of atoms before being deflected.

- Furthermore, an extremely pure crystal of copper has a very low resistance if it's really cooled down....and the atoms are all still there.

What are the Electrons Bouncing off?

- Not the atoms!
- An extremely pure crystal of copper has a very low resistance if it's really cooled down….
- This is the clue: they are deflected by thermal vibrations of the lattice—resistance increases with temperature.
- The electrons also bounce off impurities, but can pass through a pure cold lattice like light through glass… electrons are really waves!

Temperature Dependence of Resistivity

- Resistivity of metals increases approximately linearly with temperature over a wide range.
- The formula is:

$$\rho_T = \rho_0 \left[1 + \alpha \left(T - T_0 \right) \right]$$

ρ_0 being the resistance at some fixed T_0, and α the temperature coefficient of resistivity.

- An old incandescent (not LED) bulb has a tungsten wire at about 3300K, and α = 0.0045, from which

$\rho_T \approx 15 \rho_0$ not far off proportional to temperature.

Clicker Question

- What is the resistance of a 12V, 36 Watt headlight bulb?

A. 3 ohms
B. 4 ohms
C. 0.3 ohms

Clicker Answer

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- Power of 36W = IV, V = 12 so I = 3. Then $I = V/R$.

Clicker Question

- Assume the 12V, 36 Watt headlight bulb has a tungsten filament. What is its approximate power output in the first instant it is connected, cold, to the 12V battery? ($\rho_T \approx 15\rho_0$).

A. 36W

B. 2.4W

C. 540W

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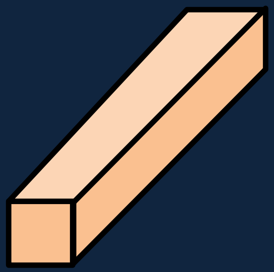
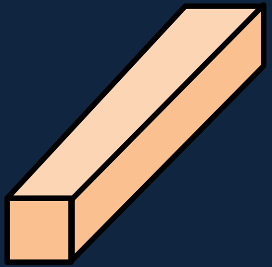
Power $P = IV = V^2/R$: R when initially cold is 1/15 of R at operating temperature of 3300K.

Drift Speed



- Take a piece of copper wire, say 1mmx1mm cross section, 1m long carrying 5 amps.
- This is 1cc of Cu, about 10 gms, about 10^{23} conduction electrons (one per atom), about 15,000C of electron charge.
- Therefore, at 5 amps (C/sec) it takes 3000secs for an electron to drift 1m.
- <u>Bottom line</u>: the drift velocity is of order 10^{-4} m/sec. (it's linear in current, and depends on wire thickness for given current, obviously.)

Drift Speed and Electron Speed



- Take a piece of copper wire, say 1mmx1mm cross section, 1m long carrying 5 amps: this wire has resistance $R = \rho\ell / A \approx 0.02\Omega$ so from Ohm's law $E \approx 0.1 \text{ V/m}$.
- This field will accelerate the electrons, $ma = eE$, approximate accn = 2×10^{10} m/s^2. This reaches drift velocity in about 0.5×10^{-14} seconds, that must be time between collisions.
- Electron speed (from quantum mechanics) is about 2×10^6 m/s, so goes of order 10^{-8} m between collisions—past dozens of atoms.

AC and DC

- Batteries provide direct current, DC: it always flows in the same direction.
- Almost all electric generators produce a voltage of sine wave form:

$$V = V_0 \sin 2\pi ft = V_0 \sin \omega t$$

- This drives an alternating current, AC,

$$I = \frac{V_0 \sin \omega t}{R} = I_0 \sin \omega t$$

and power

$$P = VI = I^2 R = I_0^2 R \sin^2 \omega t = \left(V_0^2 / R \right) \sin^2 \omega t$$

AC Average Power and rms Values

- The AC power $P = \left(V_0^2 / R\right)\sin^2 \omega t$ varies rapidly ($\omega = 2\pi f$, f = 60 Hz here), what is significant for most uses is the average power.
- The average value of $\sin^2 \omega t$ is ½.
- Define V_{rms} by $V_{\text{rms}} = \sqrt{\overline{V^2}} = V_0 / \sqrt{2}$
- Then the average power $\overline{P} = V_{\text{rms}}^2 / R$

average value of $\sin^2 \omega t$ must equal average value of $\cos^2 \omega t$. and remember $\sin^2 \omega t + \cos^2 \omega t = 1$

The standard 120V AC power is V_{rms} = 120V.

So the *maximum* voltage V_0 on a 120V line is 120x√2 = 170V!

Why Bother with AC?

- Because, as we'll discuss a little later, it's very easy to transform from high voltage to low voltage using transformers.
- This means very high voltages can be used for longer distance transmission, low voltages for local use.

Clicker Question

- The resistivity of aluminum is 58% higher than that of copper.
- A copper high voltage line has diameter 1 cm. If is replaced by an aluminum line of the same resistance, the aluminum line has diameter:

A. 1.58cm
B. 1.27cm
C. 0.8 cm
D. O.64 cm

Clicker Answer

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Remember $R = \rho L/A$. The power lines have the same length, the aluminum therefore needs 58% more cross-section area A, from which diameter up by factor $\sqrt{1.58}$.

High Voltage Power Lines …

- Are made of aluminum—you need 58% more than copper by volume, but less than half the weight, *and* it's about 65% cheaper per kg.
- No contest.
- Some steel may be added for strength.