Sound I

Physics 2415 Lecture 27

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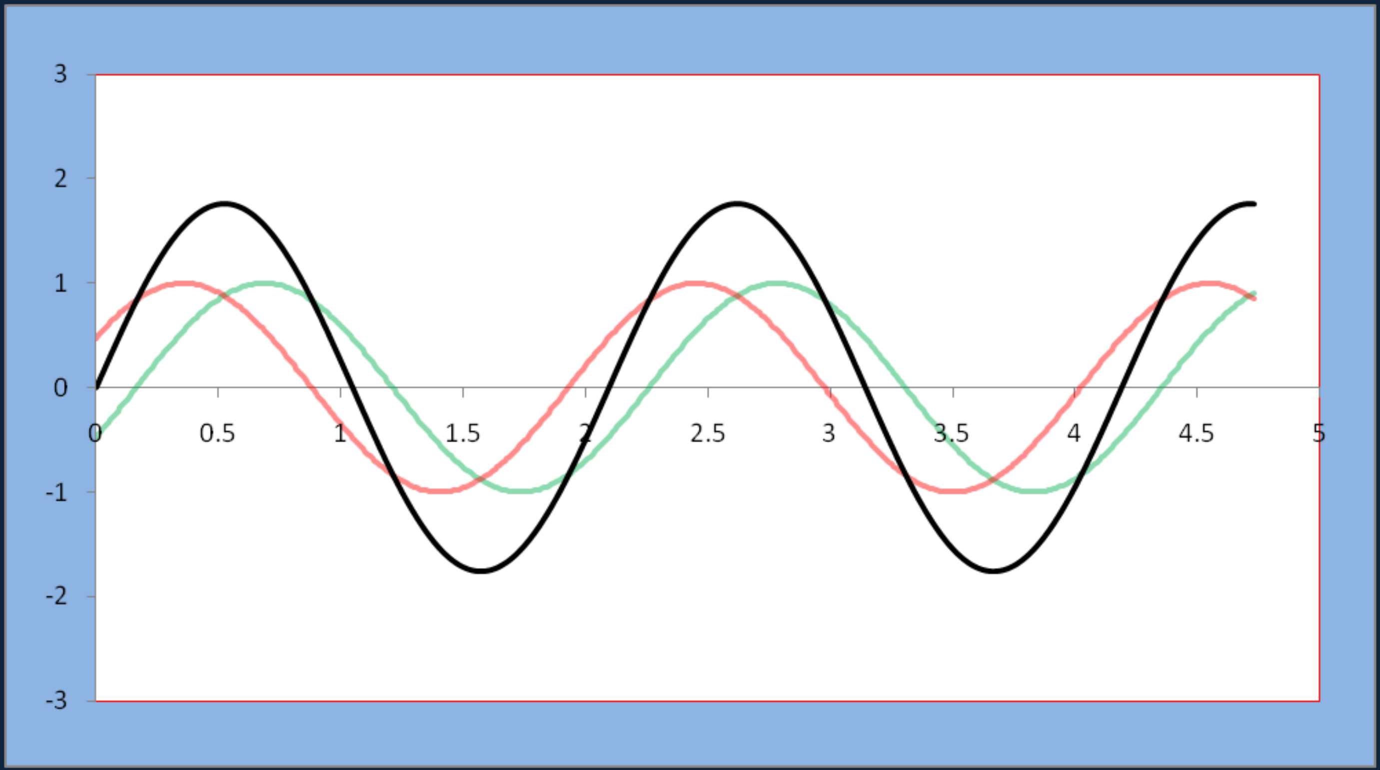
Today's Topics

- Standing waves as sums of traveling waves
- Boundary conditions
- Longitudinal waves: sound
- Amplitude and pressure variations
- Strings and pipes

Harmonic Wave Addition

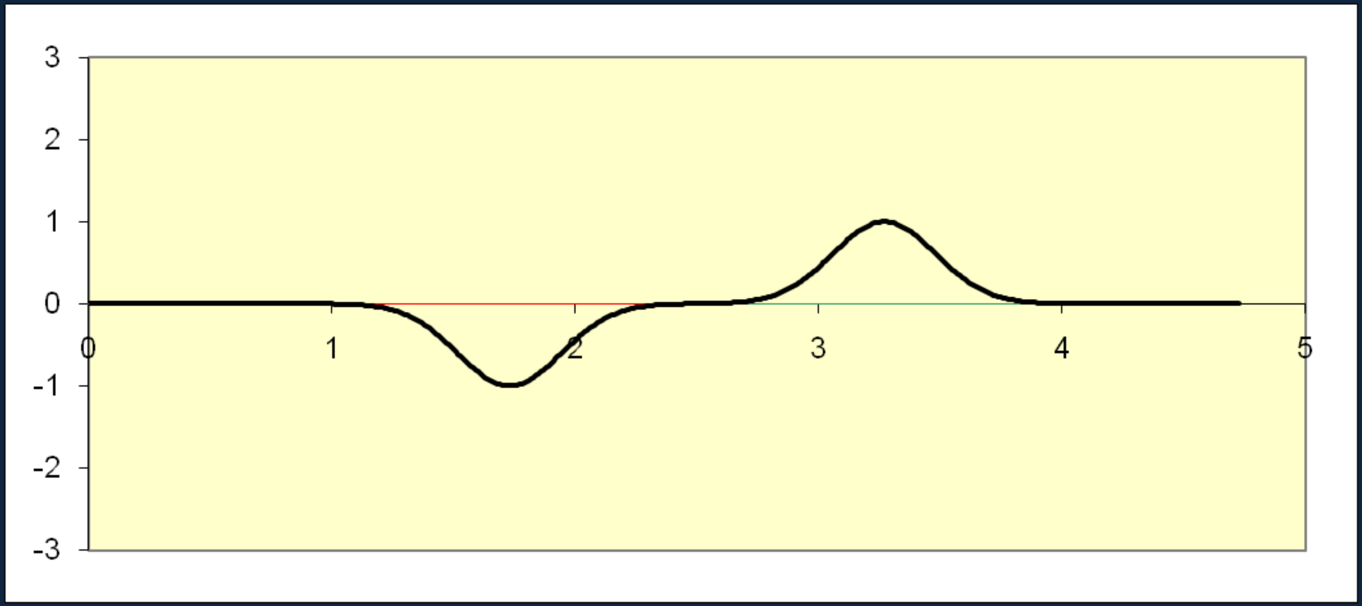
Two harmonic waves of the same wavelength and amplitude, but moving in opposite directions, add to give a standing wave.



Notice the standing wave also satisfies $\lambda f = v$, even though it's not traveling!

Pulse Encounter

It's worth seeing how two pulses traveling in opposite directions pass each other:



And here's an animation.

The (Fixed) End of the String

What happens when a pulse reaches the end of the string, and the end is fixed?

A. It will decay
B. It will bounce back, looking much the same.
C. It will bounce back, but an up pulse will become a down pulse on reflection.

The (Free) End of the String

What happens when a pulse reaches the end of the string, and the end is free? (Meaning the string is attached to a ring which can slide freely on a rod in the *y*-direction.)

A. It will decay
B. It will bounce back, looking much the same.
C. It will bounce back, but an up pulse will become a down pulse on reflection.

Strings Attached

- Suppose a black string and a less heavy red string are joined and pulled so the tensions are equal.
- A pulse is sent down the heavier black string. What happens after it gets to the join?

A. It continues with larger amplitude along the lighter red string.

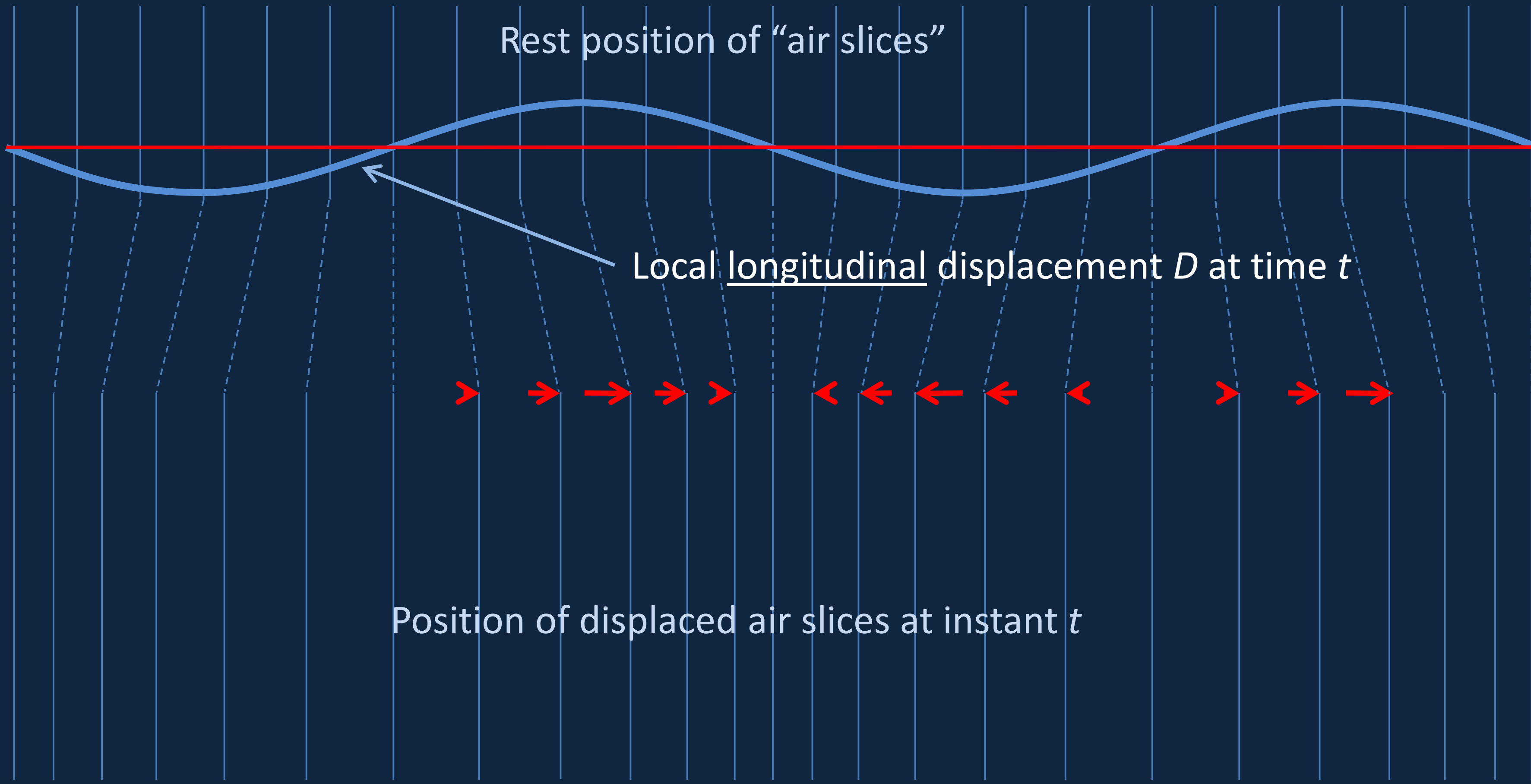
B. It part continues, part reflected with same sign.

C. Part continues, part reflected with opposite sign.

Solution

Sound Wave in a Tube

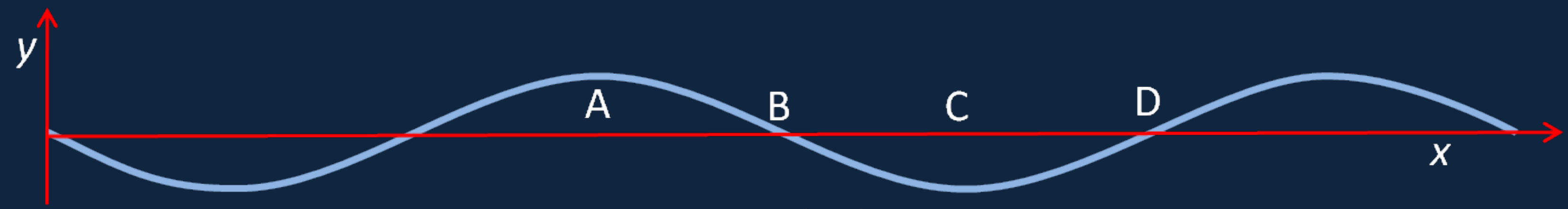
animation



Note! For the amplitude of longitudinal displacement, we'll use D instead of A.

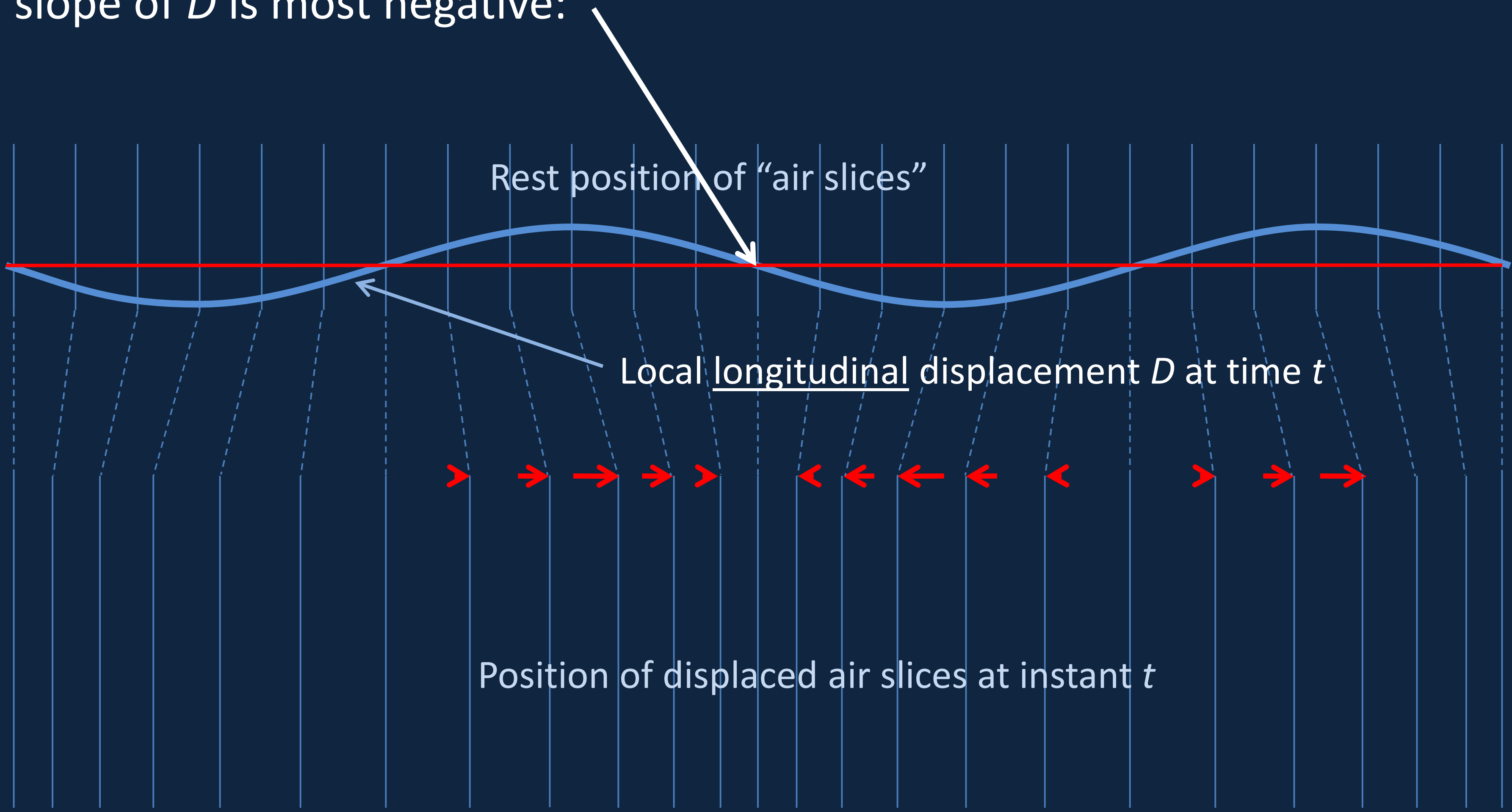
Clicker Question

For a traveling sound wave going down a tube, at the instant *t* shown below, where is the air density greatest?



y = longitudinal displacement *D*(*x*,*t*) at instant *t*

The density is greatest at B where the difference Δ*D* between the displacements sandwiching a "slice" is most negative—that is, where the slope of *D* is most negative:

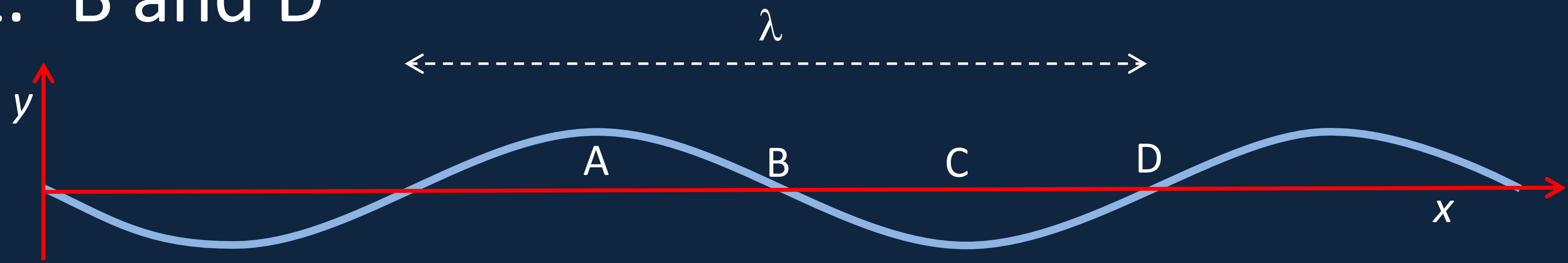


If the original (imaginary) slices have thickness Δx, the fractional change in volume as the wave passes $\Delta V/V = \Delta D/\Delta x$.

Clicker Question

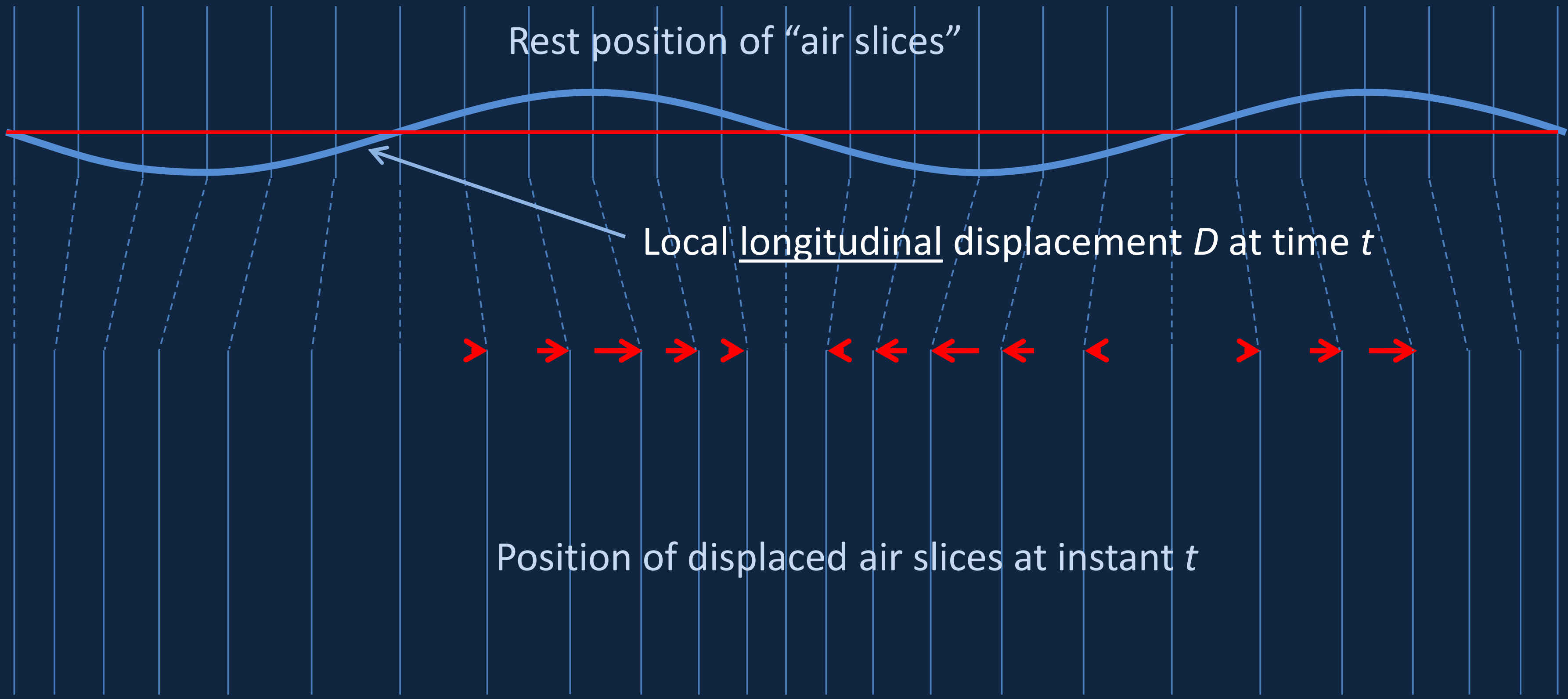
For a traveling sound wave going down a tube, at the instant *t* shown below, where in the wavelength is the pressure equal to atmospheric pressure ?

A. Only A
B. Only B
C. Only C
D. A and C
E. B and D



y = longitudinal displacement *D*(*x*,*t*) at instant *t*

The pressure variation from atmospheric (rest) pressure is given by $\Delta P = -B(\Delta V/V) = -B(\Delta D/\Delta x)$, so $\Delta P = 0$ where D has zero slope as a function of x: that is, $\Delta P = 0$ where the amplitude of D is largest: at A and C.



If the original (imaginary) slices have thickness Δx, the fractional change in volume as the wave passes $\Delta V/V = \Delta D/\Delta x$.

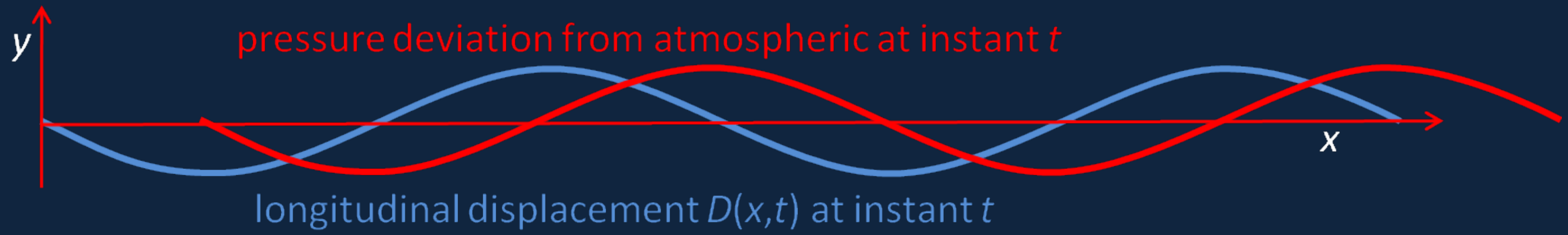
Amplitude and Pressure in a Harmonic Wave

- We found the pressure deviation from rest (atmospheric) pressure in a "slice" to be:

$$\Delta P = -B\frac{\Delta V}{V} = -B\frac{\Delta D}{\Delta x} = -B\frac{\partial D}{\partial x}$$

the last expression comes from taking the limit of very thin slices.

- So for $D = A\sin\left(kx-\omega t\right)$ we have $\Delta P = -BAk\cos\left(kx-\omega t\right)$.



Power and Loudness of Sound

- A sound wave delivers energy to any surface that absorbs it, the unit of power is watts per square meter of area perpendicular to the direction of the wave.
- Experimentally, the least power the human ear can detect is about $I_0 = 10^{-12}$ watts/m^2, the most (without pain!) is about 1 watt/m^2.
- With this vast range, we must measure power I on a logarithmic scale: we define the decibel dB by

$$\beta \text{ (in dB)} = 10 \log_{10} \frac{I}{I_0}$$

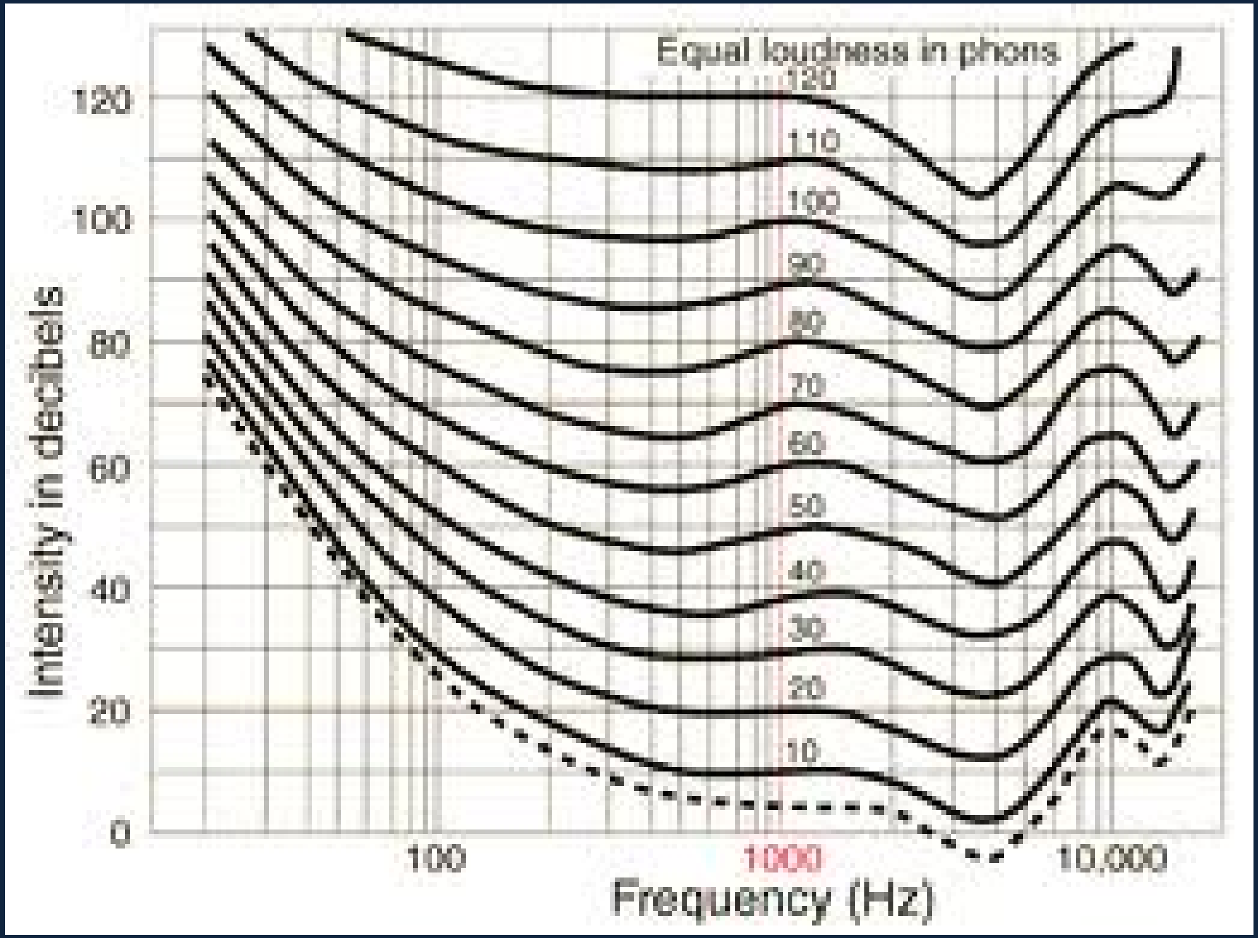
Listening Far Away…

- In the open air, the power from a source of sound radiates outward in a hemisphere, so twice as far away it's moving through a surface four times larger.
- This means power attenuates with distance as the inverse square,

$$I \propto \frac{1}{r^2}$$

Note: If a significant fraction of the energy is in *surface* waves, such as in an earthquake, for that fraction the power goes down only as $1/r$.

How Loud Does It *Sound*?



The phon is a unit of loudness: it's defined as equal to the dB at 1000Hz. (But it's not SI nor official US. It's somewhat subjective, curves vary.)

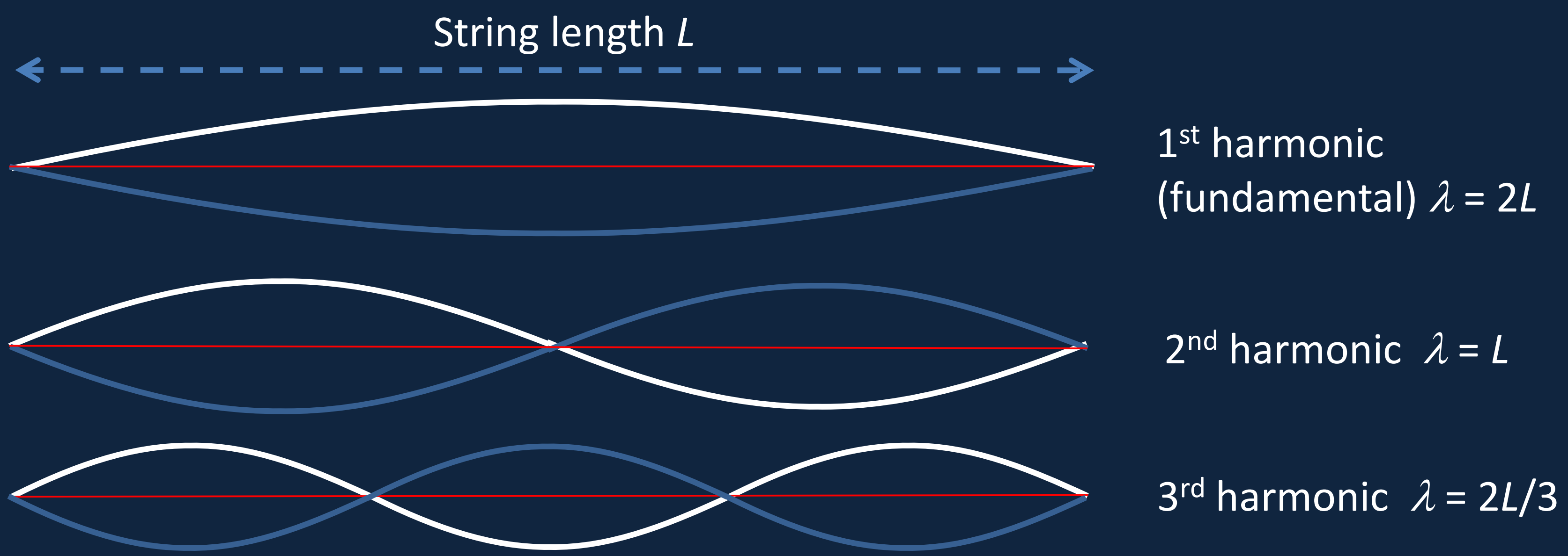
To make a 20Hz signal sound as loud as a 1000Hz signal takes a million times the power!

Amplitude of a Sound Wave

- The expression for power/unit area delivered by a sound wave is the same as a string, with the string mass/unit length simply replaced by density/m^3 (since this is mass/m for sound traveling down a pipe with cross-section 1 sq m).
- Power/sq m is sound intensity: $I = 2\pi^2 v \rho f^2 A^2$
- At the threshold of hearing at 1000Hz the amplitude is of order 10^{-11}m, $\Delta P \sim 10^{-10}$ atm.

Harmonic String Vibrations

- Strings in musical instruments have fixed ends, so pure harmonic (single frequency) vibrations are sine waves with a whole number of half-wavelengths between the ends. Remember frequency and wavelength are related by $\lambda f = v$!

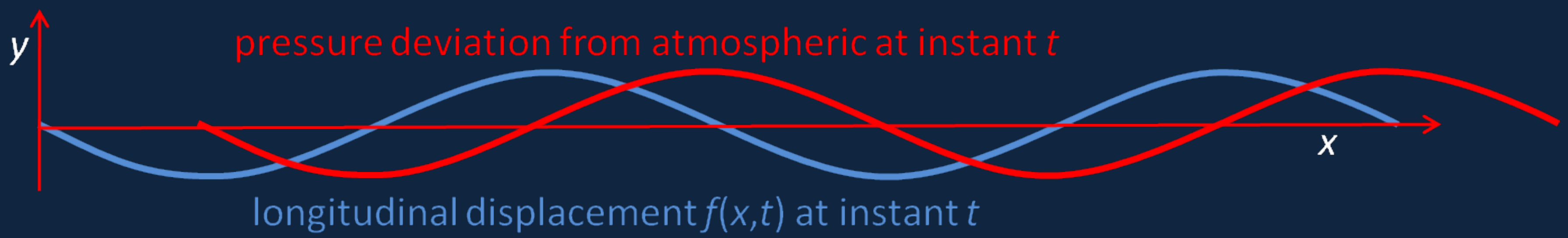


Longitudinal Harmonic Waves in Pipes

- What are possible wavelengths of standing harmonic waves in an organ pipe?
- Unlike standard string instruments, organ pipes can have two different types of end: closed and open.
- Obviously, longitudinal vibrations have no room to move at a closed end: this is the same as a fixed end for a transversely vibrating string.
- But what does the wave do at an open end?

Boundary Condition at Pipe Open End

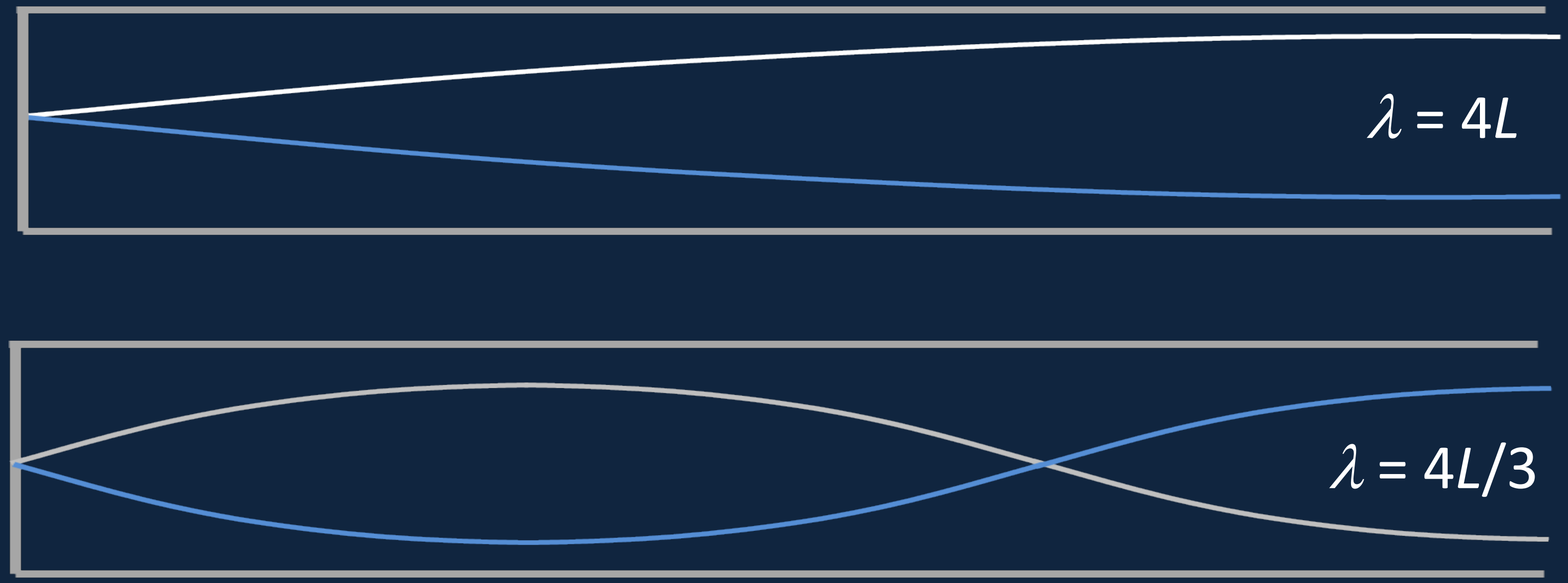
- At an open end of a pipe, the air is in contact with the atmosphere—so it's at atmospheric pressure.
- The boundary condition at the open end is that the pressure is constant, that is, $\Delta P = 0$.



- This means the amplitude of longitudinal oscillation is at a maximum at the open end!

Harmonic Modes in Pipes

- One end closed, one open:



Clicker question: what is the next value of λ?

A. L B. $4L/5$ C. $2L/3$ D. $L/2$

Clicker Answer

- $\lambda = 4L/5$:

- Both ends open: fundamental has $\lambda = 2L$.