Physics 2415: Lecture #2: Coulomb's Law, Field Lines

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Coulomb's Law

Using the two small hanging spheres, we can even find just how the attraction varies with distance, by measuring the angle the string makes with the vertical and doing a simple calculation for varying distances. This is tricky, though—the charge slowly leaks away, especially in summer, moisture in the air dampens the surfaces slightly, and they conduct.

In an experiment essentially equivalent to this, Coulomb in the 1780's established that the electrostatic force decreased with distance as the inverse square, exactly like gravity (but of course it's much stronger!). He also found the force to be proportional to the magnitude of the charge. He accomplished that by using a charged sphere, then removing it and putting it in contact with an identical but uncharged sphere, so the charge would be equally shared. Now putting the sphere back, he found the force had been halved.

The Unit of Charge

In his honor, the unit of charge is called the coulomb. The charge on the electron is $1.6\text{x}10^{-19}$ coulombs. The coulomb is *a practical unit for dealing with batteries* and electric currents, it's the amount of charge flowing down a wire per second when the current is one ampere (we'll discuss these units in detail later). Unfortunately, though, in electrostatics we *never* deal with charge on this scale, and the microcoulomb is more typical.

For Electrostatics, this is an Immense Unit of Charge…

To picture the strength of electrostatic repulsion, imagine taking an ounce of water, and imagine you could pull all the electrons off the atoms, and put them is a separate glass. (Of course, this can't be done—as you'll see!) Now put our glass of electrons and your glass of nuclei one thousand kilometers apart, say here and Orlando. What's the strength of the attractive force between them?

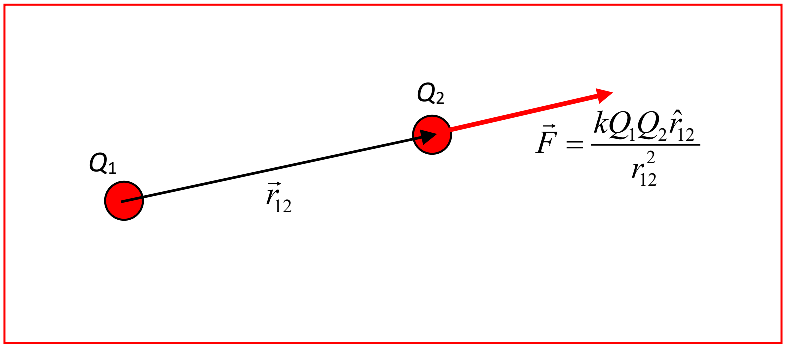
Now Avogadro's number of molecules, $6\text{X}10^{23}$, weigh a gram mole, that's 18 grams for water, so an ounce, 28 grams, is about 10^{24} atoms, or 10^{25} electrons (a water molecule has ten electrons total). One electron has charge $1.6\text{x}10^{-19}$ coulombs, so we have $Q = 1.6\text{x}10^{6}$ coulombs. Here to Orlando $r = 10^{6}$ meters, so

$F = kQQ/r^2 = 9\text{x}10^9\text{x}(1.6\text{x}10^6)^2/10^{12}$, about $2\text{x}\ 10^{10}$N … 2,000,000 tons.

From this, we can definitely conclude that for charged spheres repelling each other, the *imbalance* in electron numbers from neutral is very, very small: this is why typical electrostatic charges are *micro*coulombs, but *total* electron charge in a sphere is of order *mega*coulombs—the imbalance is of order 10^{-12} or so.

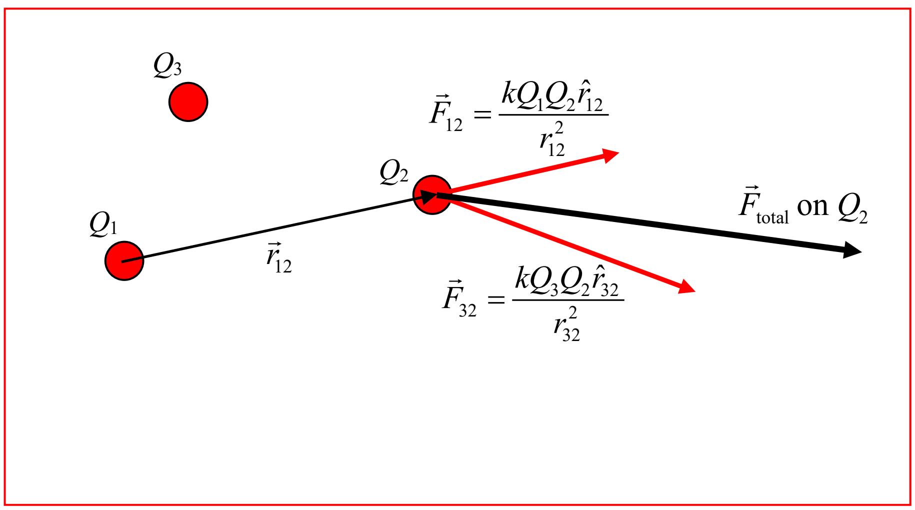
Coulomb's Law in Vector Form

We follow standard practice in denoting a vector of unit length parallel to the vector $\vec{r}$ by $\hat{r}$:



The Principle of Superposition

Electric force vectors *add*: if charge Q_2 is repelled by charge Q_1 and charge Q_3, the total repulsive force on it is the vector sum of the separate repulsive forces:



This may look obvious, but must be experimentally verified—the Law of Superposition is in fact not true for nuclear forces!

The Electric Field

Just why a charge can affect the motion of another charge some distance away is rather mysterious, as indeed is the gravitational attraction between two masses. Einstein was the first to realize that a mass distorts space time in such a way that other masses, instead of moving at constant velocity, accelerate. The earth's gravitational field slightly distorts space and time here so all masses free to fall accelerate downwards at the same rate (ignoring other forces, such as air resistance, of course).

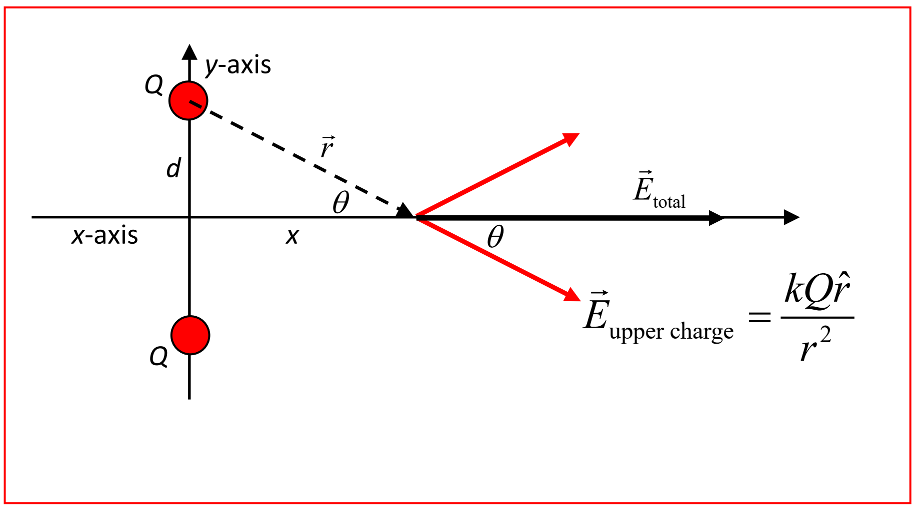
An electric charge does not distort spacetime, but does have a surrounding energy density in space, called its electric field. Another charge placed in this field experiences the inverse-square force.

The electric field $\vec{E}(\vec{r})$ at a point $\vec{r}$ is defined by stipulating that the force $\vec{F}$ on a tiny test charge q at $\vec{r}$ is equal to $q\vec{E}$.

Therefore, the electric field at a point can be determined experimentally without knowing the details of the placement of charges producing the field. It may not be necessary to know that. The reason for taking a small test charge is that if the field is partially from charges on conductors, introducing a large test charge will change the distribution of the charges on conductors, so changing the field being measured. However, if the field is from point charges, or charges on insulators, any size test charge will be fine.

Field from Two Equal Charges

Two charges *Q* are placed on the *y*-axis, equal distances *d* from the origin up and down. What is the electric field on the *x*-axis, and where does it reach a maximum value?



It's clear from the diagram that at any point on the *x*-axis, the sum of the electric fields from the two charges is itself along the axis, and has value

$$E = \frac{2kQ}{r^2}\cos\theta = \frac{2kQx}{r^3} = \frac{2kQx}{\left(x^2+d^2\right)^{3/2}}.$$

Notice that at large distances (*x* >> *d*) this goes to $2kQ/x^2$, the same as a charge 2*Q* at the origin; but actually at the origin the electric field is zero: the two components are equal and opposite. For small *x*, the field strength increases linearly with *x*. Clearly, if the field with increasing x first increases then finally decreases, it must have a maximum value somewhere. To find where this is, we use *dE*/*dx* = 0 at that point. Routine differentiation of the above expression gives the value $x_{E_{\max}} = d/\sqrt{2}$.

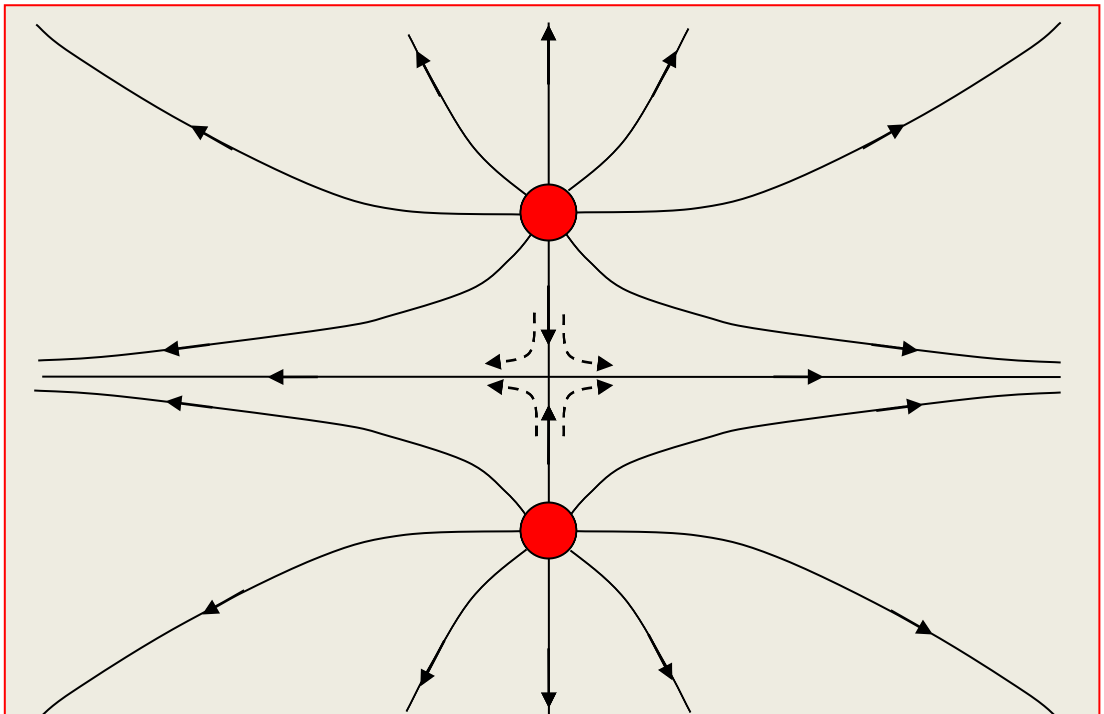
Lines of Force

One way to visualize the electric field is to draw lines of force. These are lines drawn so that at any point on the line, the electric force on a positive test charge is in the direction of the line.

Exercise: Try sketching the lines of force for the two equal charges in the diagram above.

We already know the x-axis is a line of force, since the field everywhere on it is along the axis, but in different directions for positive and negative x. We also know that anywhere on the y-axis, the force is along the y-axis, pointing away from the nearer charge. We know that close to one of the charges, its force will dominate, so the field lines initially come out close to radially from the charge. Finally, far away the two charges look like one, so again the field lines will be radial at large distances.

Here's a sketch:



Lines of force for two equal positive charges: the field is zero at the central point

To get a better idea of electric fields in this and other cases, go to this website. Sometimes attempts are made to indicate the strength of the field by how close together the lines are drawn. This is easy to see for the simplest case of a single charge, although even there one would think from a diagram on paper that the field was decreasing as $1/r$, because that's how the lines thin out—but really the field is in **three** dimensions, so actually they would thin out faster, as $1/r^2$, in a (more realistic) 3D model. The other problem with this approach is that where the field is weak, no lines appear at all, so it's difficult to figure out what's going on. We've added some dotted local lines of force near the midpoint, the place where the field strength goes to zero.

Field on the Axis of a Ring of Charge

Given what we've just done, this is very easy: if a ring has total charge Q, uniformly spread around the ring, it can be replaced by a large number of small pairs of charges arranged as in the above example, and the axis of the ring is the x-axis in the above picture for all these pairs of charges, the electric fields of all the pairs add, they're all along the axis for a point on the axis, so the answer has to be, for a ring of radius d,

$$E = \frac{kQx}{\left(x^2 + d^2\right)^{3/2}}.$$