home

Physics 2415 Lecture 8: Capacitance

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Introduction: Charging a Sphere; Definition of Capacitance

A capacitor is a device for holding electrical charge. Of course, any electrically isolated macroscopic object can hold some charge, but the term *capacitor* is only used for conductors, so the whole object is raised to the same potential when the charge is added.

Perhaps the simplest example of a capacitor is a conducting sphere of radius R. As we found earlier, a charge Q on the sphere generates an electrical field outside the sphere of magnitude $E=\left(1/4\pi\varepsilon_0\right)\left(Q/r^2\right)$, so the potential at the surface of the sphere $V=\left(1/4\pi\varepsilon_0\right)Q/R=Q/C$ with

$$C=4\pi\varepsilon_0 R.$$

That is, the charge Q *of the sphere is linearly proportional to the voltage* V, *and the coefficient* $Q/V=C$ *is termed the capacitance.*

In our system of units, the charge is measured in coulombs, and the capacitance which is raised in potential by one volt if one coulomb of charge is added is called a one farad capacitor, in honor of Michael Faraday. This is a pretty big sphere: recall $1/4\pi\varepsilon_0=9\times10^9$, so if $C=4\pi\varepsilon_0 R=1$, we have $R=9\times10^9\,\mathrm{m}$, more than ten times the radius of the Sun!

If we need to store significant quantities of charge, spheres are not the best way to go (although a sphere *is* used in the van der Graaff machine).

Parallel Plates

Far more common are capacitors made of parallel plates of conductors: in the simplest case, two flat plates of area A are placed parallel a distance d apart, where d is much smaller than the linear size of the plates. This configuration was discussed in detail in lecture 5, so we'll just take the results from there. We take it that d is sufficiently small that the field between the plates is uniform, and the field outside the plates from the charge on the plates is negligible.

When connected to a battery, one plate to the positive and one to the negative terminal, charge flows on to the plates in equal (but of course opposite sign) amounts: if charge Q flows to the positive plate, it has charge density $\sigma=Q/A$, giving a uniform electric field outwards from each side $E=\sigma/2\varepsilon_0=Q/2A\varepsilon_0$. This is the field from the positive sheet only, the field between the sheets has an equal contribution from the negative sheet, so

$$E=Q/A\varepsilon_0.$$

The voltage difference between the plates is then

$$V = Ed = Qd / A\varepsilon_0.$$

It follows immediately from the definition of capacitance, $V = Q/C$, that

$$C = \varepsilon_0 A / d$$

for the parallel plate capacitor.

Capacitance of a Coaxial Cable

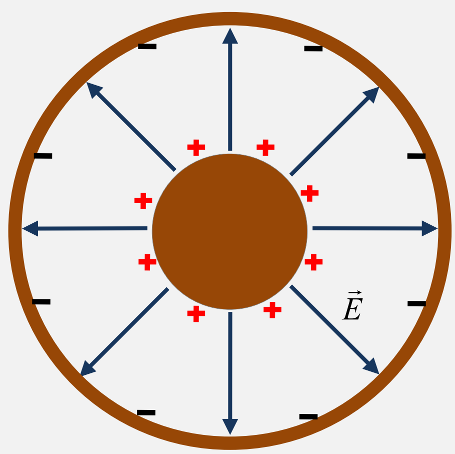
Recall from lecture 5 (where this diagram appears) the field configuration in a coaxial cable: the electric field strength between the inner solid copper wire and the outer encasing copper cylinder is given by $E(r) = \lambda / 2\pi r\varepsilon_0$, from Gauss' Law, where λ is the charge per meter on the wire (and the cylinder, of course). The voltage difference between the two cylinders is therefore, from a simple integration

$$V = \int_{r_1}^{r_2} E(r)\,dr = (\lambda / 2\pi\varepsilon_0)\ln(r_2 / r_1)$$

so the capacitance of a length ℓ is

$$C = Q/V = \ell\lambda / (2\pi\varepsilon_0)\ln(r_2/r_1)$$
$$= 2\pi\varepsilon_0\ell / \ln(r_2/r_1).$$

As we shall see later, this is important in analyzing the transmission of electromagnetic waves in coaxial cables—and that's the way the signal gets to your TV.



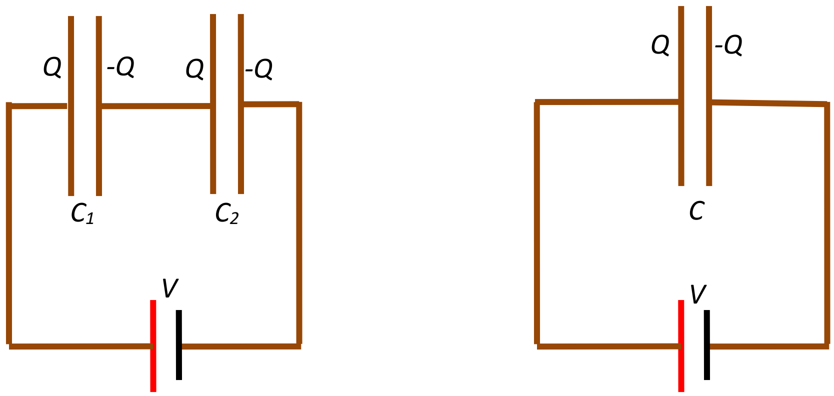
Typical electric field configuration in a coaxial cable, usually a copper cylinder and a central copper wire. The charge is on the **outside** surface of the **inner** conductor, the inside surface of the outer conductor.

Capacitors Big and Small

With parallel plates, we don't need a capacitor bigger than the Sun to get one farad. But it still has to be pretty big, if we keep the gap between plates an easily visible size, say 0.1mm. The reason is that ε_0 is so small ($8.85\text{x}10^{-12}$). The area has to be of order square kilometers! Traditional commercial capacitors lessen the gap by having plates separated by a thin layer of insulator (which is also a dielectric—see later) and roll up the plates into a many layered roll. Still, it's difficult to get much above millifarads this way in a compact capacitor. A real breakthrough came some years ago with the realization that aluminum oxidizes almost immediately on exposure to air, that the oxide layer that forms is about a micron (10^{-6} meters) thick, and is a good insulator. Capacitors were then made by putting conducting paste on to oxidized aluminum. The paste was one plate, the aluminum metal the other. More recently, capacitors have been manufactured with a layer of insulator a few atoms thick. This is another factor of 1,000 down in thickness. At the same time, the area has been vastly increased by using activated carbon, a solid which is actually many tiny granules pressed close, but with most of their surface still exposed, to give hundreds of square meters of surface in an ordinary size jar (your lungs have a similar structure—and comparable surface area, necessary to absorb oxygen at the required rate). The only drawback is that the insulating layer cannot resist more than three volts or so, this being the typical voltage to excite an atom. However, these new capacitors are measured in kilofarads, and will soon be competitive with conventional batteries in hybrid cars. One advantage over batteries is the rapidity with which capacitors can absorb and deliver power.

At the other end of the scale, dynamic rapid access memory (DRAM) in computers stores information in millions of capacitors of microscopic size, arranges in rows and columns on a chip. These are measured in femtofarads (10^{-15} farads). So capacitors are currently being manufactured over a range of sizes 10^{18}!

Combining Capacitors in Circuits: Series and Parallel



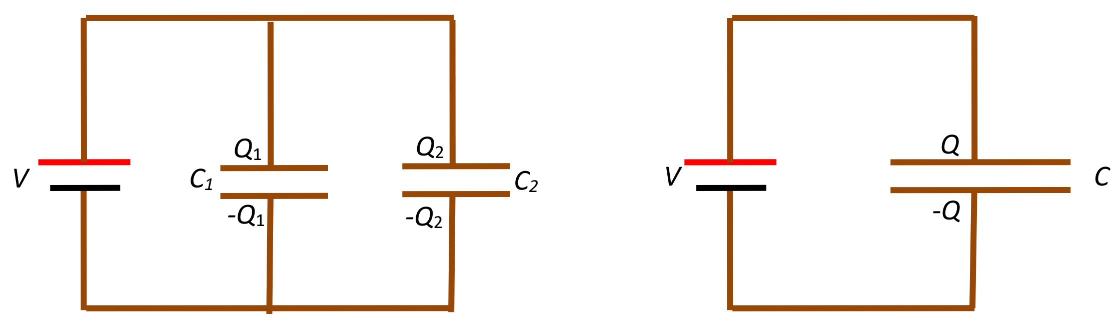
Two capacitors C_1, C_2 in **series** and the equivalent single capacitor C. The charges on C_1, C_2 must be the same, since the charge on the plate of C_1 not connected to the battery must come from C_2. Since the single capacitor C behaves identically, for given V it also draws charge Q.

Two capacitors that appear one after the other in a circuit, as shown above, are said to be in **series**. They can be replaced by a single capacitor which will behave identically, meaning if the two were in a black box with just the wires coming out the side, by testing with various voltages and noting the charge flowing in, you wouldn't be able to tell. But, given C_1, C_2 what is the value of the equivalent capacitor C? The key is to note that if the $C_1 + C_2$ combination is subject to the same external voltage as the single C, the same charge must flow in—otherwise, the C wouldn't be equivalent. Also, equally important, in the combination the Q's on the two capacitors must be the same, since the Q from the battery on C_1 will draw $-Q$ from C_2 as shown.

Now consider the total voltage drop on going around the circuits. For C, it's $V = Q/C$. For $C_1 + C_2$, there is voltage drop across each capacitor, so the total $V = Q/C_1 + Q/C_2$. These voltage drops for the two circuits are equal, so for *capacitances in series,*

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2},$$
$$C = \frac{C_1 C_2}{C_1 + C_2} \quad \text{(series)}.$$

For *capacitances in parallel,* at given voltage V, the total charge drawn from the battery by the two capacitors, $Q_1 + Q_2$ must equal the charge Q drawn by the single equivalent capacitor, from which



Two capacitors C_1, C_2 in *parallel* and the equivalent single capacitor C. The **voltages** across C_1, C_2, C are all V. Since the single capacitor C and the parallel pair C_1, C_2 behave identically, the total charge $Q_1 + Q_2$ drawn from the battery must equal Q.

$$C = \frac{Q}{V} = \frac{Q_1}{V} + \frac{Q_2}{V},$$
$$C = C_1 + C_2 \quad \text{(parallel)}.$$

Simple Picture of Adding Two Capacitors

Suppose we take two capacitors which are physically parallel metal plates: the capacitances are $C_1 = \varepsilon_0 A_1 / d_1, C_2 = \varepsilon_0 A_2 / d_2$. First, take two for which $d_1 = d_2$. Place them side by side, and connect the two top plates, then the two bottom plates: put them in parallel. Obviously, the combined capacitor C has the same $d = d_1 = d_2$, and $A = A_1 + A_2$, so $C = C_1 + C_2$. Next, take two having $A_1 = A_2$ and put them in series: For the combined C, $A = A_1 = A_2$, $d = d_1 + d_2$, the result follows.