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Physics 2415 Lecture 15: Magnetism II

Michael Fowler UVa

Force on an Electric Charge Moving in a Magnetic Field

We've already discussed the experimentally well-established force on a current element in a magnetic field, recall that for an increment $\overrightarrow{d\ell}$ of current-carrying wire it was $\vec{F} = I\overrightarrow{d\ell} \times \vec{B}$.

If the linear charge density in the wire is λ coulombs/meter, and the charge is moving at $\vec{v}$ along the wire, then the force $\vec{F}$ on the charge q in the increment $\overrightarrow{d\ell}$ of wire, $q = \lambda d\ell$, is (using $I = \lambda v$)

$$\vec{F} = \overrightarrow{Id\ell} \times \vec{B} = \lambda d\ell \vec{v} \times \vec{B} = q\vec{v} \times \vec{B}.$$

(Of course, $\overrightarrow{d\ell}$ and $\vec{v}$ are parallel vectors.)

This, then, is the force on a charge q moving at velocity $\vec{v}$ in a magnetic field $\vec{B}$.

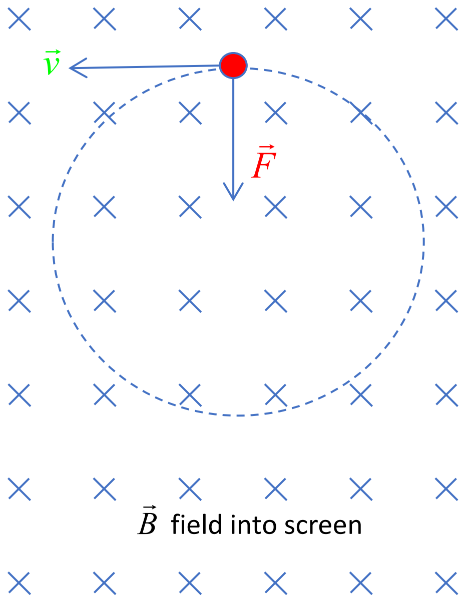
The force is *perpendicular to the direction of motion* at all times, so can do no work:

$$\vec{F} \cdot d\vec{s} = \vec{F} \cdot \vec{v}dt = q\left(\vec{v} \times \vec{B}\right) \cdot \vec{v}dt = 0.$$

Exercise: Suppose you have two parallel long wires carrying identical currents. They will attract each other, and accelerate towards each other. If the magnetic force can't do any work, how does this happen?

Hint: take the simplest possible picture—think of electrons going down a super pure metal wire, so the only constraint is that they stay in the wire by bouncing off the sides as they move down. How will switching on a magnetic field alter this picture?

Motion in a Uniform Magnetic Field



First, if the particle is moving parallel to the magnetic field it will feel no force and so continue at constant velocity.

Second, if it initially moving perpendicular to the uniform magnetic field, it will feel a sideways force proportional to its speed and will move in a circle, say radius $\vec{r}$, force $q\vec{v}\times\vec{B}$ pointing towards the center:

$$qvB = \frac{mv^2}{r}.$$

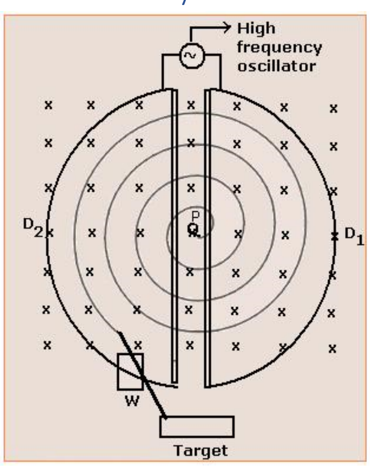
It follows immediately that the time for one circle is

$$T = 2\pi r / v = 2\pi m / qB$$

independent of the size of the circle!

This independence makes the cyclotron accelerator possible.

Proton in a Cyclotron



The two "D"s are hollow D-shaped metal boxes, open along the straight part.

The circling protons go back and forth.

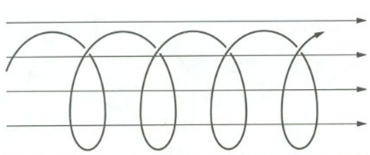
The oscillator alternates the relative voltages of the D's, so as a proton goes from one to the other it is attracted and accelerates, going into a larger, faster circle—*but with the same period*—each time.

If the proton reaches *relativistic* speeds, its mass increases and the circling time changes, recall $T = 2\pi r / v = 2\pi m / qB$.

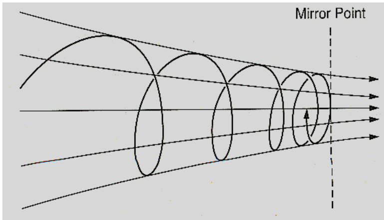
Still, in 1939 a 60-inch cyclotron at Berkeley accelerated deuterons to 16 Mev, and this was used in secret in the Manhattan Project to bombard Uranium and produce the Plutonium used in the "Fat Man" bomb, not declassified until 1948.

The relativistic mass circling time can be handled by having an oscillator with gradually decreasing frequency to match the mass increase. This is a synchrocyclotron: the problem is that now the particles must move in a tight group, whereas in the cyclotron particles could be fed in continuously.

Charged Particle in a Magnetic Field

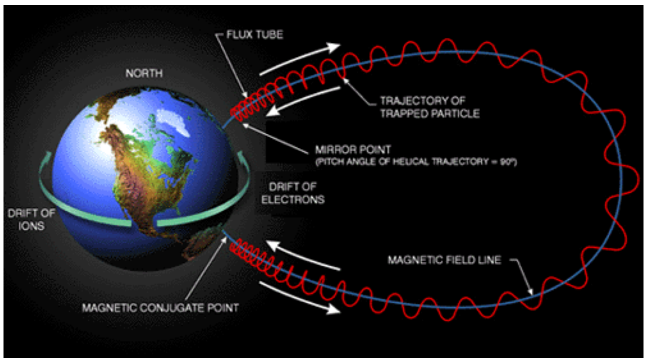


If the initial velocity is not perpendicular to the field, the motion in constant field will be circular plus a constant velocity parallel to the field—a helix.



If the field is becoming stronger in the direction of motion, the helix gets tighter, and finally reverses. This is a *magnetic mirror*, used to confine plasmas in prototype fusion reactors. The slope of the field lines gives a “backward” component to the magnetic force.

Large-Scale Magnetic Confinement



The van Allen radiation belts are filled with charged particles moving between two magnetic mirrors created by the Earth’s magnetic field. The outer belt is mostly electrons, the inner one mostly protons.