Physics 2415 Lecture 16: Magnetism III

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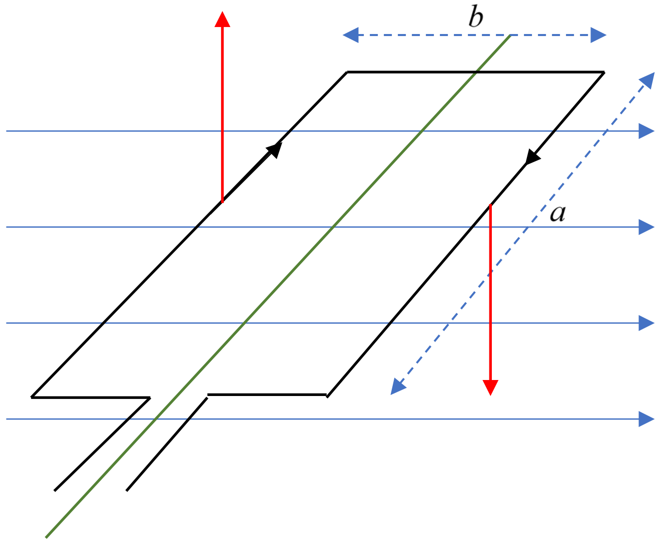
Torque on a Current Loop

This is the driving force for most electric motors, and, acting in reverse, the current generator for dynamos. It is also the basis for almost all pre-digital measuring devices: voltmeters, ammeters, etc.

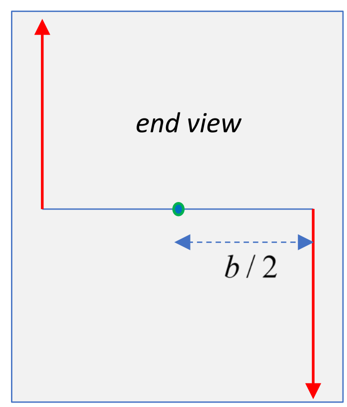
We begin with an $a \times b$ rectangular loop, horizontal, in a uniform magnetic field with field lines parallel to the end sides of the loop.



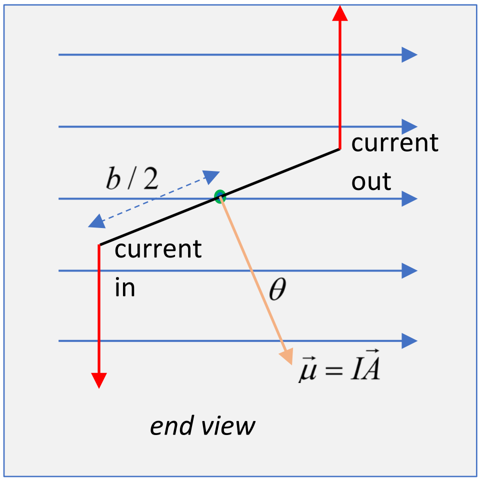
The forces on the other sides are vertical as shown, with magnitude $\left| I\vec{\ell} \times \vec{B} \right| = IaB$, and torque about the axis:

$$\tau = IaBb/2 + IaBb/2 = IabB = IAB$$

where $A = ab$ is the area of the loop.



Exercise: This formula (in terms of the loop area) works for any flat loop, not just rectangular. Try proving it!



Current Loop at an Angle

Note: for a coil with *N* turns, just multiply the single-loop result by *N*.

The current loop has a magnetic field resembling that of a short bar magnet, we define the *direction* of the loop area vector $\vec{A}$ (perpendicular to the wire loop) as that of the semi equivalent bar magnet, the magnitude of the vector $\vec{A}$ being the area.

Generalizing the result of the previous section to the case where the area vector $\vec{A}$ is no longer parallel to the magnetic field, the torque becomes (see figure)

$$|\vec{\tau}| = IAB\sin\theta,$$

meaning the loop has a dipole moment

$$\vec{\mu} = I\vec{A}$$

and as usual

$$\vec{\tau} = \vec{\mu} \times \vec{B}.$$

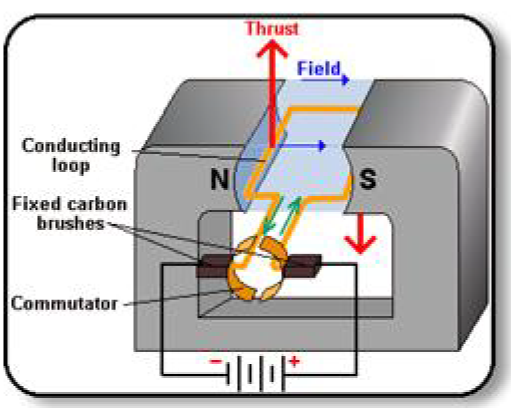
Note that the formula $\vec{\mu} = I\vec{A}$ is good for any flat loop.

Current Loop Potential Energy as Function of Angle to Field

The work done in turning the loop through incremental angle $d\theta$ is $\tau d\theta$, so, taking the zero of potential energy to be at $\theta = \pi/2$, the potential energy at arbitrary θ is the work needed to get there,

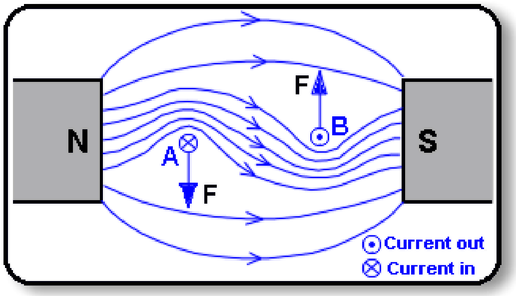
$$U = \int \tau d\theta = \int IAB\sin\theta d\theta = -\mu B\cos\theta = -\vec{\mu}\cdot\vec{B}.$$

Basic Electric Motor: the Commutator



It's just the loop in a magnetic field again, but with one crucial addition: the *commutator*.

As the loop rotates (envision it as a short bar magnet attracted by the poles of the big magnet) the commutator *switches the current direction* (notice it's made of two half-circles) and therefore switches the loop's poles, so that the loop always feels a torque in the same direction (or zero), and continues to rotate.

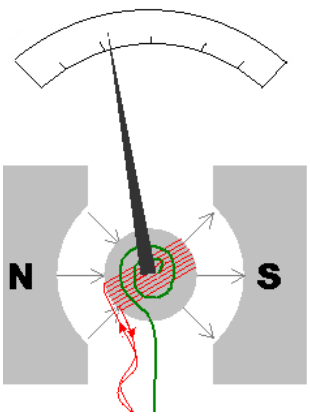


As we've discussed earlier, Faraday pictured the magnetic field lines as elastic, naturally trying to shorten themselves (and also repelling each other sideways): this helps explain the force.

Exercise: Sketch the magnetic fields from (1) the permanent magnet and (2) the current in the wire independently, then see how adding them gives a picture like this.

To see a really simple motor, click here.

Galvanometer



The galvanometer measures the torque on a small coil in a magnetic field by balancing it against a curly spring (see figure). The coil is wound around an iron core to concentrate the field, and also to keep the coil in the same strength field when it turns within the angular limits of the instrument.

Trivia: Ampère named the galvanometer in honor of Galvani, the first person to detect a current, using frogs' legs (they twitched), years before the magnetic field from a current was detected with a compass.

Predigital voltmeters and ammeters are essentially all galvanometers. In the ammeter the current to be measured goes directly through the instrument. In contrast, the voltmeter is wired between two points to detect their potential difference by letting a very small current through the meter, so as not to impact the system significantly.

Thomson's Measurement of *e*/*m* for an Electron

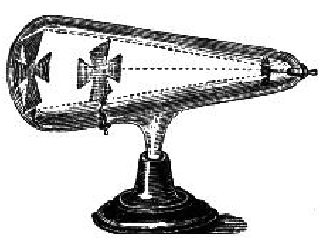


Fig. 515 Linear propagation of cathode rays; shadow formation

We discussed Thomson's experiment in lecture 11: electrons emitted by a heated wire cathode are accelerated by a high voltage in a vacuum tube and strike a phosphor-coated screen, leaving a shadow of any object (the anode here) in the way.

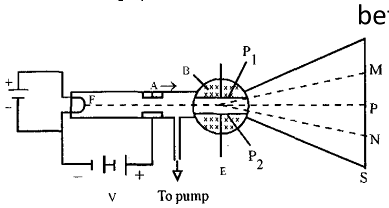


Fig. 9.2 J.J. Thomson's Experiment

Thomson narrowed the cathode rays to a pencil, which then passed between parallel charged plates (like a capacitor) P_1, P_2 (see figure) creating an area of uniform vertical electric field E. At the same time, current-carrying vertical coils were placed on each side of the tube to provide a uniform horizontal *magnetic* field B perpendicular to the ray direction, in the same region.

On entering the space between the plates, moving at speed v, the electron will be subject to a total vertical force, electric + magnetic, of

$eE + evB$. Adjusting plate voltage or coil current until the electron goes through *without deviation*, its speed will be given by

$$v = E / B.$$

We know the original accelerating voltage V, and $eV = \frac{1}{2}mv^2$, so, having found the velocity v, we can now find e/m.

Millikan's Oil Drop Experiment

To measure just the charge of an electron, it doesn't work to balance the electric force with a magnetic force, both depend on the charge. It is necessary to balance the force eE with a *nonelectrical* force. The obvious candidate is gravity, and for the force on a single electron, we need the gravitational force on a small object. Millikan (at Chicago, 1908) chose the tiny oil drops emitted by a mist spray bottle, used for example for perfume. The cloud of mist generated takes some time to settle under gravity, because the weight of a (spherical) small drop is essentially balanced by the viscous friction air resistance as it falls. For a drop of radius r, the air resistance at speed v is $6\pi r\eta v$ where η is the air's viscosity (known), so measuring v and using $6\pi r\eta v = \frac{4}{3}\pi r^3 \rho g$ (ρ the oil density) the radius can be found, and hence the weight.

The procedure, then, is to generate a small cloud of spray, then use a microscope to find a drop falling at an easily measurable rate, and thus calculate its radius and hence its weight.

(*Aside: this was evidently more accurate than measuring the size of the drop by observation. Discuss*.)

Next, a vertical electric field is turned on and adjusted until the oil drop stops falling, becoming stationary. At this point, the weight is balanced by the electrical force qE so q can be found. The experiment is repeated many times, and it is found that the measured charge is always a whole number times a basic unit: $q = ne$, n an integer and $e = 1.6 \times 10^{-19}$ coulombs. This then is the electron charge.

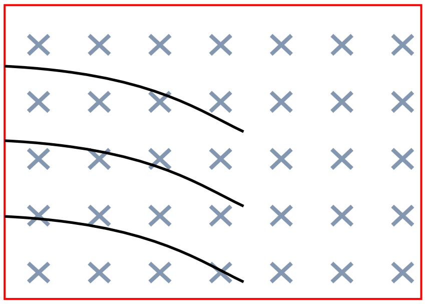
More recent update: After the invention of quark theories in the seventies, millions of dollars were spent repeating Millikan's work to search for free quarks (meaning not inside another particle), which would have charges one-third or two-thirds the electron charge. No free quark was ever found.

Exercise: Taking the oil density to be approximately that of water, what is the radius of a droplet in balance with one excess electron in a field of 1000 volts/meter? Look up air's viscosity to find its approximate rate of descent if the electric field is switched off, using the (Stokes') viscosity drag formula given above.

Hall Effect

The Hall effect was discovered by a Johns Hopkins graduate student, Edwin Hall, in 1879. He asked himself whether the force felt by a current-carrying wire in a magnetic field was a force on the wire or on the current. Remember this was before the discovery of the electron, so the concept of the electrical current was pretty vague.

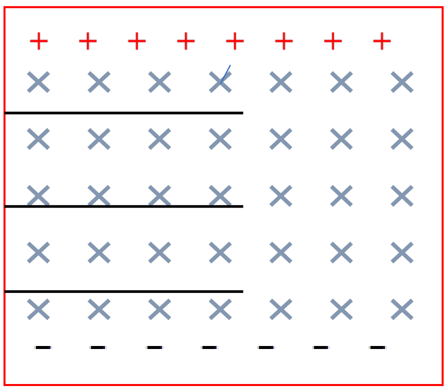
The essence of the Hall Effect can be understood with a simple model. Suppose with zero magnetic field charged particles are fed into the left-hand side of a conductor and a horizontal electric field keeps them moving to the right. (We're looking at an *average* of many particles, a single particle will follow a complex path with many collisions, see Drude.)



Next we switch on a magnetic field perpendicular to the screen, pointing inwards. The particles will experience a force $q\vec{v}\times\vec{B}$ and for negatively-charged electrons this will deviate them downwards. But this pattern won't last long: negative charge will pile up along the bottom edge, generating a repulsive electric field which will eventually exactly compensate the magnetic force, this is called the Hall field, and written

$$E_H = v_\mathrm{d} B$$

where v_d is the average, or drift, speed of the charged particles.



The Hall voltage, or emf, is the potential difference between the top and bottom of the strip. For width w, this is

$$\mathcal{E}_H = E_H w = v_\mathrm{d} B w.$$

Exercise: It was not known at the time of this experiment if the current was negative particles moving to the right or positive particles moving to the left. But the experiment could distinguish between these models. Explain why.

Mass Spectrometer: Velocity Selection and Identification

In the above discussion of Thomson's experiment to measure *e*/*m*, for a stream of particles moving in the x-direction at speed v, if there is an electric field of strength E in the y-direction and a magnetic field of strength B in the z-direction such that $\vec{E}+\vec{v}\times\vec{B}=0$ there will be no net force and the stream of particles will not be deviated. Note that this condition does not depend on the mass or the charge of the particles.

This means that if we send a stream of different kinds of particles, different masses, charges, velocities, down a narrow tube with these sideways electric and magnetic fields, only those with speed E/B will get through, the others will deviate and hit the sides.

Once we have a stream of particles all at the same velocity, we direct them into a perpendicular magnetic field (now no electric field) and they will circle with radial acceleration $v^2/r = qvB/m.$ If we detect them after half a circle, the half circle path radius will be proportional to the particle mass, so we can read off the proportion of different particle masses in the stream: this is termed Mass Spectrometry.

One use of mass spectrometry is carbon dating. Most elements have several isotopes: the nuclei of course have the same number of protons, but different numbers of neutrons. The isotopic ratio can change with time if one of the isotopes is radioactive and so decays. This is the case with the isotope carbon-14, continually produced from nitrogen (and cosmic radiation) in the upper atmosphere, but when absorbed into living tissue it decays with a half-life around 5700 years, so its fraction is good for dating in the range 500 – 50,000 years.

Mass spectrometry also works for molecules, an example being drug detection in a urine sample.