home

Physics 2415 Lecture 17: Sources of Magnetic Field I

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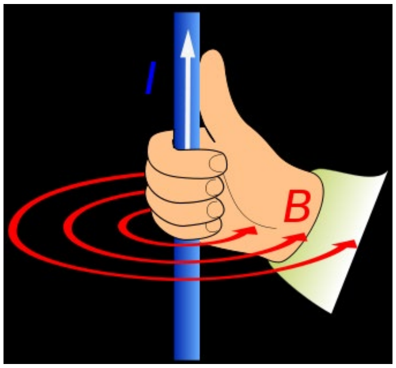
Magnetic Field from a Current in a Long Straight Wire



From many experiments, the lines of magnetic force are circles around the wire, direction determined by the right-hand rule.

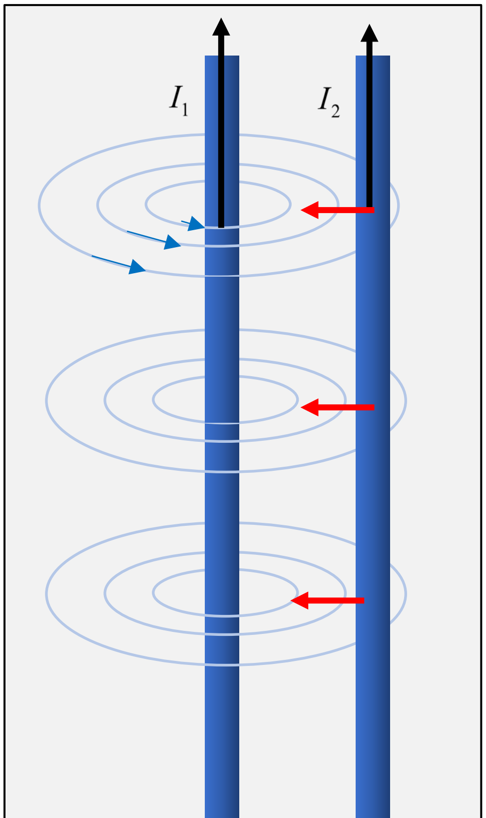
The field *strength* is proportional to the current, and inversely proportional to distance from the wire.

$$B=\frac{\mu_0}{2\pi}\frac{I}{r},\quad \mu_0\cong 4\pi\times 10^{-7}\text{ Tesla.m/A.}$$

μ_0 Update

In an earlier version of these notes, we wrote $\mu_0=4\pi\times 10^{-7}$ exactly, and that was true at the time—but things have changed. From 2019 on, the speed of light, the electron charge and Planck's constant have been given specific numerical values (effectively, this defines the units of length, etc., the unit of time having been defined in terms of certain frequencies of the Caesium atom) and the result is that μ_0 changed from its previously defined value by about one part in ten billion. So for this course, and likely the rest of your life, you can stick with the old value—but be aware of this trivium.

Force Between Parallel Wires



Showing field from I_1, force on I_2.

The field from current I_1 is $B = \frac{\mu_0}{2\pi}\frac{I_1}{r}$, circling the wire, and the current I_2 will feel a force $I_2 \vec{\ell} \times \vec{B}$ per length ℓ, so the force per meter on wire 2 is

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

towards wire 1 and wire 1 will feel the opposite force.

Bottom line: ***Like currents attract.***

Definitions of the Ampere and the Coulomb

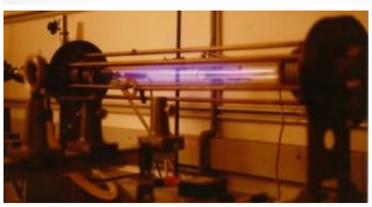
The traditional definition of the ampere, the unit of current, is based on the two-wire scenario above, two long equal parallel *one amp* currents one meter apart feel an attractive force 2×10^{-7} N/m.

The unit of charge, the *Coulomb* is the charge flow per second in a one amp current.

As mentioned above, the units have been redefined, but the change is around one part in ten billion so will not concern us.

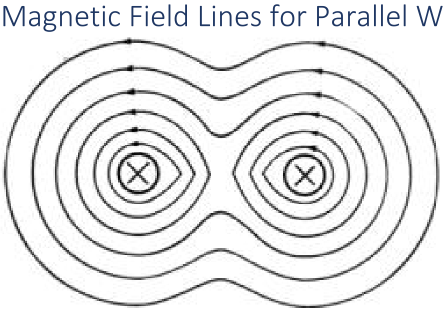
Like Currents Attracting

This is a piece of copper pipe: lightning sent a large current through it, the parallel currents attracted each other and pulled the pipe with them to a very hot central volume.



The same thing happens on sending a large current through a plasma, and intense heat is generated. This is one possible scenario for raising the temperature of small nuclei sufficiently to trigger fusion. Unfortunately, plasmas have many instabilities under these conditions, and it seems decades will still be needed to make this a practical power source.

Magnetic Field Lines for Parallel Wires

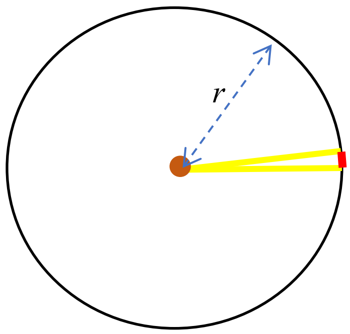


The magnetic field at a point is the vector sum of the two fields circling the wires.

Exercise: check the diagram on the left (for equal currents) by first sketching the two sets of circles (one for each wire) then use a different color for the vector sum in a few places, to see how this pattern emerges. What's going on in the middle?

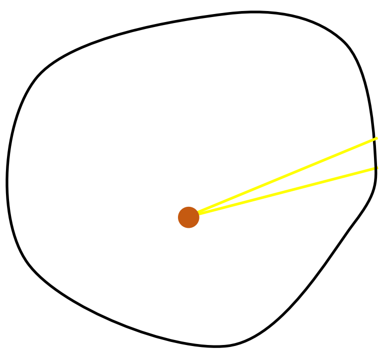
Finally, sketch the field lines if the currents are *opposite*. Make clear what happens in the middle.

Introducing Ampère's Law



Consider a current I in a long wire perpendicular to the screen, and the integral $\oint \vec{B} \cdot d\vec{\ell}$ around a circle of radius r as shown. Taking $B = \mu_0 / 2\pi r$, and $d\ell = r d\theta$ (the small red vector), the integral is just over θ from zero to 2π and

$$\oint \vec{B} \cdot d\vec{\ell} = \frac{\mu_0}{2\pi r} \oint r d\theta = \mu_0 I.$$



Suppose now we take a *noncircular* contour for the integral. The increment $\vec{B} \cdot d\vec{\ell}$ only has a contribution from the component of $d\vec{\ell}$ which is parallel to $\vec{B}$, and has length $r d\theta$, so we get the same result for the integral.

This is even still true if we draw the curve in three-dimensional space, because the added dimension (perpendicular to the screen) is perpendicular to $\vec{B}$ so makes no contribution to $\vec{B} \cdot d\vec{\ell}$.

Exercise: **Important!** Do the same exercise but with the wire *outside* the curve.

Prove the answer is *zero*. *Hint*: Track what happens to θ as you go around once, check $\oint d\theta$.

Ampère's Law

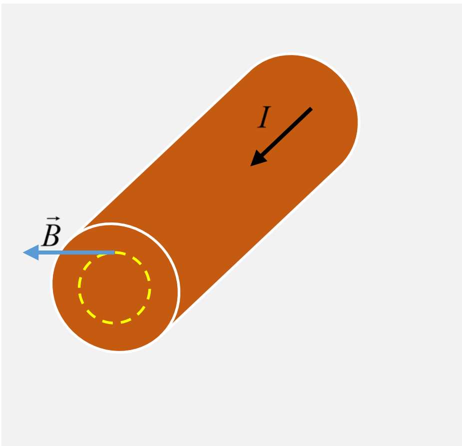
From the two cases discussed above, we can see that for a magnetic field from many long straight wires in arbitrary directions,

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}},$$

where I_{encl} counts only currents that penetrate a surface roofing the integration curve.

This is *Ampère's law*, and in fact is true for any collection of time-independent currents, well-verified experimentally. (Our "proof" above is only for a collection of straight-line currents. A general proof is not difficult, but needs a bit more calculus.)

Field Inside a Wire



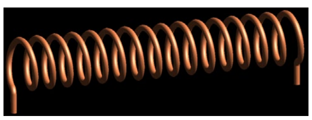
Apply Ampère's law to the dashed circular path of radius r inside the wire as shown. From symmetry (and no monopoles), the field must be tangential.

The surface "roofing" this path has area πr^2, the whole wire has cross-section area πR^2 so the current flowing through the path is Ir^2 / R^2, and Ampère's law gives

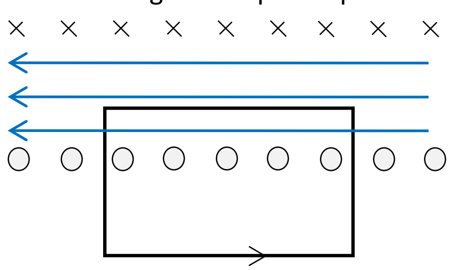
$$\oint \vec{B} \cdot \overrightarrow{d\ell} = 2\pi r B = \mu_0 I r^2 / R^2,$$

$$B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2}.$$

Field Inside a Solenoid



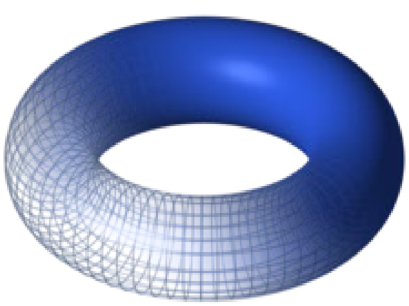
Take a rectangular Ampèrian path as shown below. Assume the external magnetic field negligible, and the field inside parallel to the axis (a good approximation for a long solenoid). For current I, n turns/meter,



$$\oint \vec{B} \cdot \overrightarrow{d\ell} = B\ell = \mu_0 n I \ell,$$

$$B = \mu_0 n I.$$

Magnetic Field of a Toroid



A toroid here is equivalent to a solenoid with the axis turned into a circle so the two ends connect. This is a promising design for containing a hot plasma, unlike the "magnetic bottle" where charged particles can escape at the ends.

Notice the current-carrying wires spiraling around the surface.

To find the field, imagine slicing the donut to get maximal flat surface area, the wires intersect this surface in two concentric circles, say currents coming up on the inner circle, down on the outer circle.

From symmetry, the lines of magnetic field must themselves be circles centered on the main axis, and the field must have the same strength all the way round.

For the integral around a circle of radius r,

$$\oint \vec{B} \cdot \overrightarrow{d\ell} = 2\pi r B = \mu_0 N I$$

where N is the number of times the wire penetrates the circular disk having the circular contour as its boundary, meaning the number of times the wire circles the contour.

If the circular contour is inside the toroid, it contains the inner circle, so $B = \dfrac{\mu_0 N I}{2\pi r}$, notice this is not uniform, unlike the linear solenoid. If the circular contour is outside the solenoid, both up currents and down currents penetrate the circular area, cancelling, and there is no field. In fact, it is easy to show there is no field except within the toroid volume.