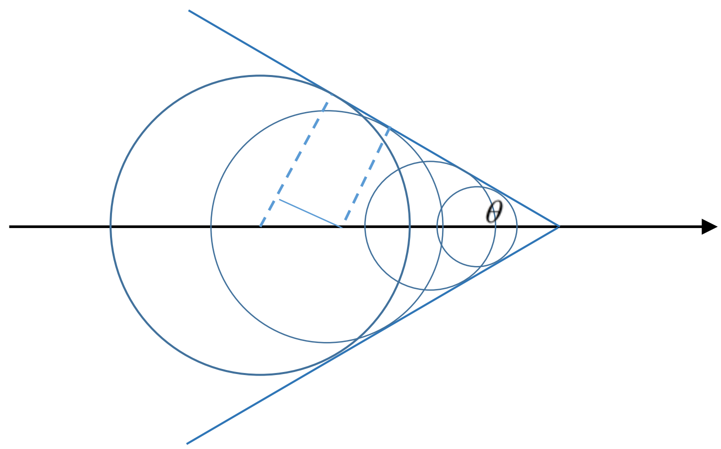
22. Cherenkov Radiation

Introduction

When a charged particle moves through a transparent medium at a speed greater than the phase velocity of light in the medium, radiation is emitted. This is called Cherenkov radiation, after Pavel Cherenkov, who was the first to detect it experimentally in 1934. He received the Nobel Prize for this discovery, shared with the two theorists, Tamm and Frank, who explained the effect. (In fact, it had been predicted by Heaviside in 1888.)

A simple picture of Cherenkov radiation is provided by comparing it with the shock wave generated by a supersonic airplane. (Check out this animation.) The sound waves emitted at each instant radiate outwards in a spherical wave in the air, the plane outstrips the outgoing wave and continues to generate spheres, the envelope of these spheres is the supersonic shock wave, a cone, with semi angle given by $\sin\theta = \dfrac{v_{\text{sound}}}{v_{\text{plane}}}$.



It's called the Mach cone.

It's interesting to note that the waves along the shock front are *in phase*: look at the path length difference between the two dashed lines, and the time variation of the emissions, a d the ratio of velocities.

Going now to the analogous problem of a charged particle moving through a dielectric material faster than the speed of light in that material, the first point to make is that the radiation loss is small, so to a very good approximation the particle is moving at constant velocity, so the particle itself is not directly responsible for the radiation, only indirectly, in that the moving charge causes a localized dielectric shielding response, obviously time-dependent, and this is the excitation that travels outwards from the immediate vicinity of the particle.

We can analyze this in the usual way, the potential and field observed at current time t at displacement $\vec{R}$ from the current particle position comes from the particle + shielding at t_{ret}, the observer being at $\vec{R}_{\text{ret}}$ relative to the particle position at that time (see drawing) so

$$t - t_{\text{ret}} = \frac{R_{\text{ret}}}{c/n}.$$

However, this is different from our earlier analysis of fields from a moving charge. Glancing at the figure above, we can see that there are points in the cone where two of the circles intersect, such a point will see two signals at the same instant from the moving charge at different earlier times! In fact, this is true for *any* point *in* the cone: there is really a continuous family of these circles, beginning with tiny ones near the point of the cone, think of the continuously expanding circles, given a point inside the cone, eventually a circle will pass through it, then it will be inside subsequent circles until it is on the boundary of one.

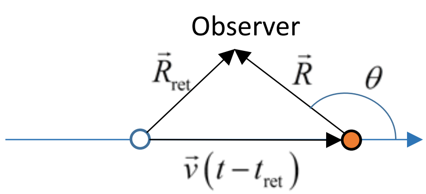
Or, in line with our previous analysis of the world line of a particle always being inside the light cone, the "light cones" in the dielectric are narrower, and a particle moving faster than the speed of light in the medium can intersect the cone twice (at a steady velocity).

The two points of intersection coincide for an observation point *on* the cone, so there is a heavy concentration of energy there.

To find the electric and magnetic fields, the Lienard-Weichert potentials still work if we replace $\varepsilon_0 \to \varepsilon,\ \mu_0 \to \mu,\ \vec{\beta}_n = \vec{v}/c_n = \vec{v}/(c/n)$. We'll assume for now that ε, μ are frequency-independent, so we can write

$$\varphi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R - \vec{\beta}_n \cdot \vec{R}}\right]_{\mathrm{ret}}, \quad \vec{A}(\vec{r},t) = \left[\frac{q\vec{v}}{R - \vec{\beta}_n \cdot \vec{R}}\right]_{\mathrm{ret}}.$$

Notice we have to put the observer inside the Mach cone, or nothing will be observed.



From the diagram, and squaring $R_{\mathrm{ret}} = \left|\vec{R} + \vec{v}(t - t_{\mathrm{ret}})\right|$, we find a quadratic equation for $t - t_{\mathrm{ret}}$, with roots

$$t - t_{\mathrm{ret}} = \frac{-\vec{v}\cdot\vec{R} \pm \sqrt{(\vec{v}\cdot\vec{R})^2 - (v^2 - c_n^2)R^2}}{v^2 - c_n^2} \geq 0.$$

Now for Cherenkov radiation, we also need $\vec{v}\cdot\vec{R} < 0, \quad \sin\theta \leq \sin\theta_C = 1/\beta_n.$ (This last being when the two roots coincide).

Zangwill finds $\varphi, \vec{A}$ by feeding in these two values, and so can find the fields. He finds the electric field indeed is singular on the shockwave front, but its nonzero value inside the cone points the wrong way—towards rather than away from the charge. There is a simple explanation for this: as the charge passes, it polarizes the medium, but it's moving so fast that immediately behind it is a net negative charge density that takes time to relax, in fact it is part of the outwardly propagating wave. However, Wikipedia currently doesn't like this picture, I'm not sure why.

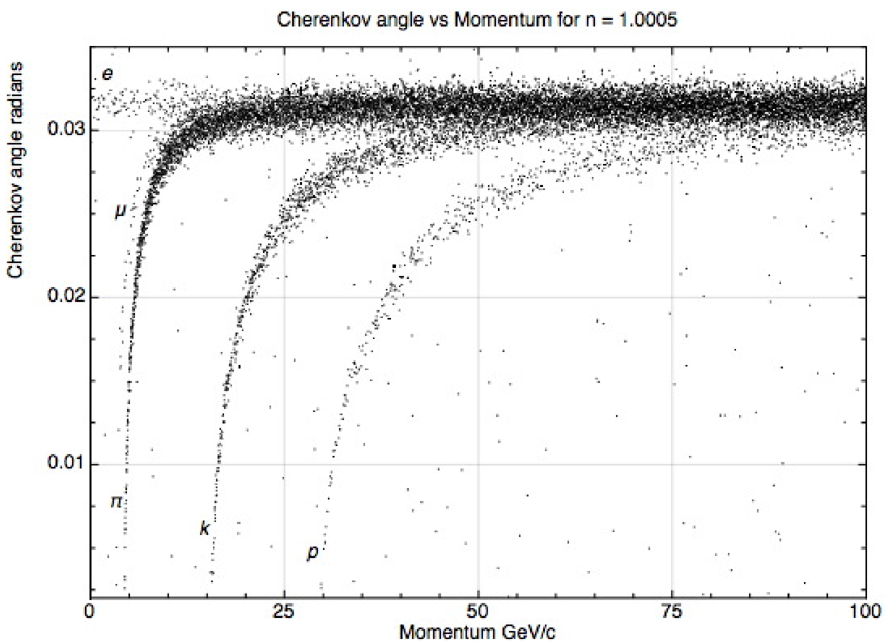
The sharp nature of the shock wave front (which is actually somewhat fuzzed out by frequency variations in dielectric response, which we've ignored) suggests that the radiation covers a wide range of frequencies, and over the visible range it tends if anything to increase with frequency, so the light appears blue.

Uses of Cherenkov Radiation

Widely used in biochemistry and medicine, radioactive markers emit energetic particles which are traveling faster than light in biomaterials (essentially, water). Cherenkov radiation is used to detect them.

Used in neutrino searches: example, IceCube, at the south pole. One cubic kilometer of ice, with many Cherenkov radiation detectors. Finds a few dozen neutrinos a year.

Used in many high energy experiments to find velocity of particles of known momentum, hence find their mass, in identifying decay fragments from collisions. The dielectric medium is typically a gas, with $\varepsilon = 1.0005\varepsilon_0$, say, so there won't be any Cherenkov radiation if the particle is traveling slower than $c/1.0005, \quad \gamma < 30$ or so.



Various Generators of Radiation

Particles accelerated in a synchrotron can be caused to radiate by going linearly through a *wiggler*. This is a sequence of alternating magnets causing the path to deviate from a straight line to a sinusoidal path. (In practice, this is a straight section in the storage "ring", clearly not a circular ring.) The consequent back and forth oscillation generates dipole radiation, the frequency determined by particle speed and magnet spacing.

The *free electron laser* (FEL) is essentially a wiggler with reflecting mirrors at the two ends, so the radiation intensity builds coherently to a strength where there is ponderomotive feedback on the particle current.

Transition radiation (TR) is generated when a fast particle goes into a material with a different refractive index. The electric field lines will be changed, and connecting the old lines to the new ones will generate radiation, analogous to Purcell's analysis of radiation from a nonrelativistic accelerating charge.

The *Smith-Purcell Effect* is the generation of radiation by a fast charged particle moving just above a metal diffraction grating. The sequential response of the strips of metal will generate outgoing coherent radiation at certain angles. The is currently used for terahertz radiation, a frequency range difficult to generate otherwise.